

Hybrid Classical-Quantum Computing: Applications to Statistical Mechanics of Neocortical Interactions

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Abstract

Several commercial quantum computers are now available that offer Hybrid Classical-Quantum computing. Application is made to a classical-quantum model of human neocortex, Statistical Mechanics of Neocortical Interactions (SMNI), which has had its applications published in many papers since 1981. However, this project only uses Classical (super-) computers. Since 2015, PATHINT, has been used as a numerical algorithm for folding path-integrals. Applications in several systems in several disciplines has generalized been from 1 dimension to N dimensions, and from classical to quantum systems, qPATHINT. Papers have applied qPATHINT to neocortical interactions and financial options. The classical space described by SMNI applies nonlinear nonequilibrium multivariate statistical mechanics to synaptic neuronal interactions, while the quantum space described by qPATHINT applies synaptic contributions from Ca²⁺ waves generated by astrocytes at tripartite neuron-astrocyte-neuron sites.

Index terms— path integral, quantum systems, neocortical interactions.

<https://quantumzeitgeist.com/tensorflow-for-quantumhits-first-birthday/> <https://www.tensorflow.org/quantum>
Two previous XSEDE project-codes are merged for this project, "Electroencephalographic field influence on calcium momentum waves" and "Quantum pathintegral qPATHTREE and qPATHINT algorithms". These Classical codes on Classical computers define a Hybrid Classical-Quantum system.

Previous fits to electroencephalographic (EEG) data have been published using quantum wave-packets defining a specific class of (re-)generated Ca²⁺ ions at tripartite neuron-astrocyte-neuron sites, which influence synaptic interactions [Ingber(2018)]. Since 2011 [Ingber(2011), Ingber(2012a)], During tasks requiring short-term memory (STM), classical as well as quantum calculations are consistent with interactions between momenta p of these wave-packets with a magnetic vector potential A generated by highly synchronous firings of many thousands of neocortical neurons. An analytic (closed-form) path-integral calculation of the quantum process, in terms of a wavefunction with expectation value p interacting with A , defines quantum interactions coupled to a macroscopic system [Ingber (2017a)].

It is important to further address this system using these to enable realistic inclusion of shocks to the wave-packet. Due to the regenerative process of the wave-packet, e.g., due to collisions between Ca²⁺ ions in the wave-packet causing some ions to leave the wave-packet during its hundreds of msec lifetime, or as new ions enter from the astrocyte processes, the wavefunction is repeatedly projected onto quantum subspaces. These may be considered as random processes [Ross (2012)]. PATHTREE/qPATHTREE and PATHINT/qPATHINT can include shocks in the evolution of a short-time probability distribution over thousands of foldings [Ingber(2016a), Ingber(2017a), Ingber (2017b)].

1 b) SMNI

Statistical Mechanics of Neocortical Interactions (SMNI) was developed in the late 1970's [Ingber(1981), Ingber(1982), Ingber(1983)] and enhanced since by fitting experimental data from short-term memory (STM)

and electroencephalography (EEG), including papers on fits to attention data [Ingber(2018)] and affective data [Alakus ~al.(2020) Alakus, Gonen, Turkoglu, Ingber(2021b)].

2 c) PATHINT

The path-integral defines code that numerically propagates short-time conditional probability distributions [Ingber ~al.(1991)Ingber, Fujio, Wehner, Wehner Wolfer(1983a), Wehner Wolfer(1983b)]. This was generalized to PATHINT in N dimensions, and applied in several disciplines ??Ingber(2000a)

3 d) qPATHINT

PATHTREE and PATHINT were generalized to quantum systems, qPATHTREE and qPATHINT [Ingber(2016a), Ingber(2017b), Ingber(2017c)].

A companion paper treats "Hybrid classicalquantum computing: Applications to statistical mechanics of financial markets" [Ingber(2021c)].

4 e) Organization of Paper

Section 2 further describes SMNI in the context of this project.

Section 3 further describes Adaptive Simulated Annealing (ASA) in the context of this project.

Section 4 further describes qPATHINT in the context of this project.

Section 5 describes how the calculation proceeds between SMNI and qPATHINT. Section 6 describes performance and scaling issues.

Section 7 gives motivation to this project. Calculations performed on the Ookami supercomputer at StonyBrook.edu test a closed-form derived solution with and without shocks. Shocks clearly lower cost functions.

Section 8 is the Conclusion.

5 f) Caveat

As stated previously in these projects [Ingber(2018)], "The theory and codes for ASA and [q]PATHINT have been well tested across many disciplines by multiple users. This particular project most certainly is speculative, but it is testable. As reported here, fitting such models to EEG tests some aspects of this project. This is a somewhat indirect path, but not novel to many physics paradigms that are tested by experiment or computation."

6 II. SMNI

Statistical Mechanics of Neocortical Interactions (SMNI) has been developed since 1981, in over 30+ papers, scaling aggregate synaptic interactions describing neuronal firings, scaling minicolumnarmacrocolumnar columns of neurons to mesocolumnar dynamics, and scaling columns of neuronal firings to regional (sensory) macroscopic sites identified in EEG studies ??Ingber(1981), ??ngber(1982), ??ngber(1983), ??ngber(1984), ??ngber(1985a), ??ngber(1994)].

Success of SMNI has been to discover agreement/fits with experimental data from modeled aspects of neocortical interactions, e.g., properties of short-term memory (STM) ??Ingber(2012a)], including its capacity (auditory 72 and visual 42), duration, stability, primacy versus recency rule ??Ingber(1984), ??ngber (1985a), ??ngber(1994), Ingber(1995), ??ngber(2000b), Ingber(2012a), Ingber Nunez(1995)], EEG dispersion relations ??Ingber(1985b)], as well other phenomenon, e.g., Hick's law [Hick(1952), ??ngber(1999), Jensen (1987)], nearest-neighbor minicolumnar interactions within macrocolumns calculating rotation of images, etc ??Ingber(1982), ??ngber(1983), ??ngber(1984), ??ngber (1985a), ??ngber(1994)]. SMNI was also scaled to include mesocolumns across neocortical regions to fit EEG data [Ingber(1997), Ingber(2012a), Ingber(2012b), Ingber (2018), Ingber Nunez(2010)].

7 a) XSEDE EEG Project

The Extreme Science and Engineering Discovery Environment (XSEDE.org) project since February 2013, "Electroencephalographic field influence on calcium momentum waves," has fit SMNI to EEG data, developing ionic Ca2+ momentum-wave effects among neuron-astrocyte-neuron tripartite synapses modified parameterization of background SMNI parameters. Both classical and quantum physics support the development of the vector magnetic potential of EEG from highly synchronous firings, e.g., as measured during selective attention, as directly interacting with these momentum-waves, creating feedback between these ionic/quantum and macroscopic scales ??Ingber(2012a)

8 i. qPATHINT for SMNI

qPATHINT includes quantum regenerative process to Ca2+ wave-packets as reasonable shocks to the waves that typically do not damage its coherence properties. A proof of principal has been published [Ingber(2017b)].

9 b) SMNI With A

A model of minicolumns as wires supporting neuronal firings, largely from large neocortical excitatory pyramidal cells in layer V (of I-VI), gives rise to currents, in turn giving rise to electric potentials measured as scalp EEG [Ingber(2011), Ingber(2012a), Nunez et al.(2013)Nunez, Srinivasan, Ingber]. This gives rise to a magnetic vector potential $\mathbf{A} = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} dV'$ (1)

Note the log-insensitive dependence on distance. In neocortex, $\mu_0 \approx 4\pi \times 10^{-7}$ H/m, where 0 is the magnetic permeability in vacuum = $4\pi \times 10^{-7}$ H/m (Henry/meter), where Henry has units of kg-m-C⁻², the conversion factor from electrical to mechanical variables. For oscillatory waves, the magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$ and the electric field $\mathbf{E} = -\nabla \phi - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}$ do not have this log dependence on distance. Thus, A fields can contribute collectively over large regions of neocortex [Ingber(2012a)] The magnitude current is determined by experimental data on dipole moments $\mathbf{Q} = |\mathbf{I}|z \hat{\mathbf{z}}$ where $\hat{\mathbf{z}}$ is the direction of the current $\mathbf{I}$ with the dipole spread over z . $\mathbf{Q}$ ranges from 1 pA-m = 10^{-12} A-m for a pyramidal neuron [Murakami Okada(2006)], to 10⁹ A-m for larger neocortical mass [Nunez Srinivasan(2006)]. Currents give rise to $\mathbf{A} \approx 10^{-28}$ kg-m/s. The velocity of a Ca²⁺ wave can be 20-50 μ m/s. In neocortex, a typical Ca²⁺ wave of 1000 ions, with total mass $m = 6.655 \times 10^{-23}$ kg times a speed of 20-50 μ m/s, gives $p \approx 10^{-27}$ kgm/s. This yields p to be on the same order as $\mathbf{A}$.

10 i. Results Including Quantum Scales

The wave-function $\psi(\mathbf{r}, t)$ of the interaction of A with p of Ca²⁺ wave-packets was derived in closed form from the Feynman representation of the path integral using path-integral techniques, not including shocks [Schulten(1999)], modified here to include $\mathbf{A} \cdot \nabla \psi$ (??) = ? $\int \psi^* \nabla \psi \cdot \mathbf{A} dV$?? = ? $\int \psi^* \nabla \psi \cdot \mathbf{A} dV$ (2) $\psi = \frac{1}{\sqrt{2\pi}} \exp\left\{i\left[\frac{1}{2} \int \psi^* \nabla \psi \cdot \mathbf{A} dV + \frac{1}{2} \int \psi \nabla \psi^* \cdot \mathbf{A} dV\right]\right\} \exp\left\{i\left[\frac{1}{2} \int \psi^* \nabla \psi \cdot \mathbf{A} dV + \frac{1}{2} \int \psi \nabla \psi^* \cdot \mathbf{A} dV\right]\right\}$ (2)

PATHINT also has been used with the SMNI Lagrangian ?? for STM for both auditory and visual memory [Ericsson Chase(1982), Zhang Simon(1985)], calculating the stability and duration of STM, the observed 7 ± 2 capacity rule of auditory memory, and the observed 4 ± 2 capacity rule of visual memory [Ingber(2000a), Ingber Nunez(1995)].

11 ii. Results using $\langle \mathbf{A} \cdot \nabla \psi \rangle \approx \mathbf{A} \cdot \nabla \psi$ * $\mathbf{A} \cdot \nabla \psi$

The author previously used his derived analytic expression for $\langle \mathbf{A} \cdot \nabla \psi \rangle \approx \mathbf{A} \cdot \nabla \psi$ * $\mathbf{A} \cdot \nabla \psi$ in classical-physics SMNI fits to EEG data using ASA [Ingber(2016b)] Calculations used one additional parameter across all EEG regions to weight the contribution to synaptic background ?? $\mathbf{A} \cdot \nabla \psi$. A was proportional to the currents measured by EEG, i.e., firings ?? $\mathbf{A} \cdot \nabla \psi$. The "zero-fit-parameter" SMNI philosophy was enforced, wherein parameters are selected and enforced between experimentally determined ranges [Ingber(1984)].

Sometimes Testing cost functions were less than their Training cost functions, a result sometimes found in previous studies using this data. This likely is due to great differences in data, likely from great differences in subjects' contexts, e.g., possibly due to subjects' STM strategies including effects calculated here. ASA optimizations in this project always included "finishing" ASA importance-sampling with the modified Nelder-Mead simplex code included in the ASA code to ensure best precision.

iii. Assumptions for quantum SMNI Some assumptions made for this quantum enhancement of SMNI can be determined by future experiments.

The quantum wave-function of the Ca²⁺ wavepacket was calculated, adding multiple collisions due to their regenerative processes, and it was demonstrated that overlaps with just-previous wave-functions during the observed long durations of hundreds of ms typical of Inclusion of repeated random shocks to the above wave-function $\psi(\mathbf{r}, t)$?? (??) demonstrated only small effects on the projections of the wave-packet after these shocks, i.e., the survival time was calculated [Facchi Pascazio(2008), Ingber(2018)].

Of course, the Zeno/"bang-bang" effect may exist only in special contexts, given that decoherence among particles is known to be very fast [Preskill(2015)]. Therefore, the constant collisions of Ca²⁺ ions as they enter and leave the Ca²⁺ wave-packet due to the regenerative process that maintains the wave may perpetuate at least part of the wave, permitting the Zeno/"bang-bang" effect. qPATHINT as used here provides an opportunity to explore the coherence stability of the wave due to serial shocks of this process.

12 iv. Nano-Robotic Applications

It is possible that the above considerations could lead to pharmaceutical products contained in nanosystems that could affect unbuffered Ca²⁺ waves in neocortex [Ingber(2015)]. A Ca²⁺ -wave momentumsensor could act like a piezoelectric device [Ingber (2018)].

The nano-robot would be sensitive to local electric/magnetic fields. Highly synchronous firings during STM processes could be directed by piezoelectric nanosystems to affect background/noise efficacies via control of Ca²⁺ waves. This could affect the influence of Ca²⁺ waves via the vector potential A, etc.

v. Free will Further qPATHINT calculations could give additional support to researching possible quantum influences on highly synchronous neuronal firings relevant to STM, yielding connections to consciousness and "Free Will" (FW).

As described previously [Ingber(2016a), Ingber(2016b)], experimental feedback from quantumlevel processes of tripartite synaptic interactions with large-scale synchronous neuronal firings, recognized as being highly correlated with STM and states of attention, may be established using the quantum no-clone "Free Will Theorem" (FWT) [Conway Kochen(2006), Conway Kochen(2009)].

Ca 2+ quantum wave-packets may generate states proven to have not previously existed, since quantum states cannot be cloned. In this context, these quantum states may be influential in large-scale patterns of synchronous neuronal firings, rendering these patterns as truly new. The FWT may consider these patterns as new decisions not solely based on previous decisions. These considerations are quite different and independent of other philosophical considerations, e.g., as in <https://plato.stanford.edu/entries/qt-consciousness/>.

Note that only recently has the core SMNI hypothesis since circa 1980 [Ingber(1981), Ingber(1982), Ingber(1983)], that highly synchronous patterns of neuronal firings process high-level information, been verified experimentally [Asher(2012)]

13 c) qPATHINT for SMNI

A previous project tested applications of qPATHTREE and qPATHINT. The wave-function $\delta \psi$ is numerically propagated from its initial state, growing into a tree of wave-function nodes. At each node, interaction of the Ca 2+ wave-packet, via its momentum p , with highly synchronous EEG, via its collective magnetic vector potential A , determines changes of timedependent phenomena. Changes occur at microscopic scales, e.g., due to modifications of the regenerative wave-packet as ions leave and contribute to the wavepacket, determining the effect on tripartite contributions to neuron-astrocyte-neuron synaptic activity, affecting both p and A . Such changes also influence macroscopic scales, e.g., changes due to external and internal stimuli affecting synchronous firings, and thereby A . At every time slice, quantum effects on synaptic interactions are determined by expected values of the interactions over probabilities $(\delta \psi \delta \psi^* \delta \psi)$ determined by the wave-functions at their nodes.

Due to the form of the quantum Lagrangian/Hamiltonian, a multiplicative Gaussian form (with nonlinear drifts and diffusions) is propagated. This permits a straight-forward use of Gaussian quadratures for numerical integration of the expectation of the momenta of the wave-packet, i.e., of $\langle \delta \psi \delta \psi^* \delta \psi \rangle$. E.g., see https://en.wikipedia.org/wiki/Gaussian_quadrature

14 d) Comparing EEG Testing Data with Training Data

Using EEG data from ψ due to the same currents I . Therefore, A is linearly proportional to ψ with a simple scaling factor as a parameter in fits. Additional parameterization of background synaptic parameters also are included, $\psi \psi^* \psi$ and $\psi \psi^* \psi \psi^*$.

15 e) Investigation into Spline-Laplacian Transformation

As is common practice, codes for the Spline-Laplacian transformations were applied to all electrodes measured on the scalp. However, the author thinks that the transformation should be applied to each Region of neocortex separately (e.g., visual, auditory, somatic, abstract, etc.), since each region typically participates in attention differently. This process is further tested in this project.

16 III.

17 ASA Algorithm

For parameters $\psi \psi^* \psi$ [$\psi \psi^* \psi$, $\psi \psi^* \psi$](3)

sampling with the random variable $\psi \psi^* \psi$ [1,1] $\psi \psi^* \psi = \psi \psi^* \psi + \psi \psi^* \psi$ (4)

the default generating function is $\delta \psi \delta \psi^* \delta \psi$ (5) $\psi \psi^* \psi = 1 \ln(1 + 1/\psi \psi^* \psi)$ (6)

in terms of "temperatures" for parameters [Ingber(1989)] $\psi \psi^* \psi = \psi \psi^* \psi \exp(\psi \psi^* \psi / \psi \psi^* \psi)$ (6)

The default ASA uses the same distribution for the annealing schedule for the acceptance function ψ used for the generating function $\delta \psi \delta \psi^* \delta \psi$.

The ASA default functions can be substituted with user-defined functions [Ingber(1993), Ingber (2012c)].

ASA has been applied to studies of COVID-19, fitting forms like $\psi \psi^* \psi$, for variables ψ and parameters ψ and ψ^* , in the drifts and covariances of conditional probability distributions [Ingber(2021d)].

With over 150 OPTIONS, ASA permits robust tuning over many classes of nonlinear stochastic systems. These many OPTIONS help ensure that ASA can be used robustly across many classes of systems.

18 IV.

19 Path-Integral Methodology a) Generic Applications

Many systems are defined by (a) Fokker-Planck/Chapman -Kolmogorov partial -differential equations, (b) Langevin coupled stochastic-differential equations, and (c) Lagrangian or Hamiltonian pathintegrals. All three such systems of equations are equivalent mathematically, when limits of discretized variables are taken

This defines a process fitting EEG using SMNI with qPATHINT numerically calculating the Quantum path-integral between EEG epochs. At the beginning of each EEG epoch time is reset ($t=0$); the wave-function is decohered ("collapsed") by any EEG measurement. Until the end of any EEG epoch, there are multiple calls to SMNI functions to calculate the evolution of the Classical distribution. This replaces the author's Quantum path-integral closed-form time-dependent analytic solution.

26 Performance and Scaling

27 a) Scaling Estimates

28 ii. qPATHINT

iii. Projected Hours/Service Units (SUs) for this Project nSubjects x 2 (switch Train/Test) yields a 24array set of 1-node jobs.

29 VII.

Calculations on the Ookami supercomputer at StonyBrook.edu tested the path-integral derived analytic (closed-form) expression [Ingber(2018)] for astrocyte Ca²⁺ wave synaptic interactions

ASA was used with 200000 valid generated states for optimizations, then the modified Nelder-Mead code was used to sometimes gain extra precision with 5000 valid generated states. Training and Testing sets of data were used for 12 subjects, then the Training and Testing sets were exchanged. Runs were done with the added shocks and without these shocks. Each of the 48 runs took about 2 days on the Ookami supercomputer at StonyBrook.edu .

Volume XXI Issue VI Version I The number 2561 of EEG epochs is a region of high amplitude of times from 17 to 22 secs after the tasks began, defining epochs to be about 0.002 sec.

31 Conclusion

A numerical path-integral methodology is used by SMNI to fit EEG to test quantum evolution of astrocyte-(re-)generated wave-packets of Ca²⁺ ions that suffer shocks due collisions and regeneration of free ions. SMNI is generalized with quantum variables using qPATHINT.

SMNI has fit experimental data, e.g., STM and EEG recordings under STM experimental paradigms. qPATHINT includes quantum scales in the SMNI model, evolving Ca²⁺ wave-packets with momentum p serial shocks, interacting with the magnetic vector potential A due to EEG, via the $(p + \hbar A)$ interaction at each node at each time slice t , in time with experimental EEG data.

Published pilot studies have given rationales for developing this particular quantum path-integral algorithm, to study serial random shocks that occur in many systems. This quantum version can be used for many quantum systems, which is increasingly important as experimental data is increasing at a rapid pace for many quantum systems.

<https://docs.ocean.dwavesys.com/projects/hybrid/en/latest/index.html>

<https://www.rigetti.com/what>

<https://azure.microsoft.com/en-us/solutions/hybrid-cloud-app/#overview>

<https://www.ibm.com/it-infrastructure/z/capabilities/hybrid-cloud>

These Classical

computers

optimization program in systems that are described by quantum variables using these companies' Quantum computers [

[Note: Introduction a) Hybrid Computing several commercial Classical-Quantum computers now can be accessed via the Cloud, e.g., Rigetti, D-Wave, Microsoft, and IBM [Ingber(2021a)]; see]

Figure 1:

,
Ingber(2016b),
Ingber(2018),

Ingber(2012b),
Ingber(2017b),

Ingber(2015),
Ingber(2017c),

Figure 2:

Figure 3:

¹(E)

²Year 2021 © 2021 Global Journals Hybrid Classical-Quantum Computing: Applications to Statistical Mechanics of Neocortical Interactions

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Citi ~al. (2010)Citi, Poli, Cinel, Goldberger ~al. (2000)Goldberger, Amaral, Glass, Hausdorff, Ivanov, Mark, Mietus, Moody, Peng, Stanley] <http://physionet.nlm.nih.gov/pn4/erpbci> As was done previously, fitting SMNI to highly synchronous waves (P300) during attention tasks, for each of 12 subjects, it is possible to find 10 Training runs and 10 Testing runs [Ingber(2016b)]. A region of continuous high amplitude of 2561 lines represents times from 17 to 22 secs after the tasks began. Spline-Volume XXI Issue VI Version I (E)

Figure 4:

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Note that in accordance with common practice, standalone website URLs are not necessarily referenced at the end, but often appear within the body of text. E.g., see <https://libanswers.snhu.edu/faq/8627> <https://www.scribbr.com/apa-examples/cite-a-website/> <https://libguides.css.edu/APA7thEd/Webpages>

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