Linking Teaching in Mathematics and the Subjects of Natural Science

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Abstract

Educational researchers and policy-makers have for some time touted the need for interdisciplinary teaching. However, despite this desire for a change towards interdisciplinary teaching, teachers are often uncertain about how to go about planning, implementing, and sustaining interdisciplinary teaching programs. This is partly due to the lack of a framework for integrating productive ideas across the disciplines. This paper focus on how to grasp the challenges of an interdisciplinary approach to teaching in mathematics and the subjects of natural science. Based on contemporary mathematics and science education we design a didactical framework for interdisciplinary teaching centered on modeling activities across mathematics and the disciplines of natural science. To exemplify the potential of the framework we present a case study of an intensive in-service teacher-training program for mathematics and biology teachers. The teachers were presented to the didactical framework and in pairs of two, one mathematics teacher and one biology teacher; they designed and implemented interdisciplinary mathematicsbiology teaching sequences. The teachers? reports on their development and implementation of the teaching sequences and presentations given at a final seminar show that in general it is possible for the teachers from two discplines to plan, carry through, evaluate and report about interdisciplinary modeling activities.

Index terms—

1 Introduction

isciplines are historically constructed, but in the recent decades, they exhibit a broad trend toward greater porosity of boundaries. The idea of interdisciplinary is to combine multiple disciplines into one activity. Whereas this may appear to be simple and straightforward, in practice it turns out that those participating in an interdisciplinary endeavor often find it difficult to work across traditional discipline boundaries. In an educational context interdisciplinary implies that the still dominating monodisciplinary approaches should be replaced by approaches enabling more connections to existing knowledge and thus lead to more profound and integrated learning. Despite a widespread desire for a change towards interdisciplinary teaching, teachers are often uncertain about how to go about planning, implementing, and sustaining interdisciplinary teaching programs. Understandable, as teachers traditionally have had their academic training within one or two mono-disciplinary programs. Further, there are many conceptual pitfalls pertaining to the very idea of interdisciplinary teaching and it seems that there is a Author: University of Southern Denmark. e-mail: cmich@imada.sdu.dk genuine need for a scholarly discussion about exactly how teachers could be equipped to implement fruitful interdisciplinary activities. Implementing an interdisciplinary teaching approach requires considerations at the level of discipline matter and at the level of pedagogy as well. This is far from a trivial task, partly due to the lack of a framework for integrating productive ideas from a variety of theoretical and practical perspectives. To grasp the challenges from the rise of the interdisciplinary approach to teaching there is a need for ? A didactical framework for interdisciplinary teaching activities. ? In-service teacher training with focus on interdisciplinary teaching activities. ? Prototypes of meaningful interdisciplinary teaching sequences.

In this paper, we address the potential and challenges of interdisciplinary teaching between mathematics and the disciplines of natural science in a Danish upper secondary education context. Mathematics plays a crucial role in the disciplines of role is brought about predominantly through the building, employment, and assessment of mathematical models. Galileo wrote that the book of nature is written in the language of mathematics, and his quantitative approach to understanding the natural world arguably marks the beginning of modern science. Nearly 400 years later, the teaching of mathematics and the disciplines of natural science is still mainly monodisciplinary and fragmented, and focus in mathematics teaching is on relatively specialized algebraic techniques. The challenge is to replace the current monodisciplinary approach, where knowledge is presented as a series of static facts disassociated from time with an interdisciplinary approach, where mathematics, biology, chemistry and physics are woven continuous together.

Interdisciplinary learning and teaching involving mathematics education has become of considerable interest to some mathematics educators (e.g. Sriraman and Freiman 2011), and we address this issue mainly from a mathematics education position, but we also draw on science education research literature to emphasize central coincident issues in mathematics and science education. We design a didactical framework for interdisciplinary instruction between Global Journal of Human Social Science -Year 2017 some time touted the need for interdisciplinary teaching. However, despite this desire for a change towards interdisciplinary teaching, teachers are often uncertain about how to go about planning, implementing, and sustaining interdisciplinary teaching programs. This is partly due to the lack of a framework for integrating productive ideas across the disciplines. This paper focus on how to grasp the challenges of an interdisciplinary approach to teaching in mathematics and the subjects of natural science. Based on contemporary mathematics and science education we design a didactical framework for interdisciplinary teaching centered on modeling activities across mathematics and the disciplines of natural science. To exemplify the potential of the framework we present a case study of an intensive in-service teacher-training program for mathematics and biology teachers. The teachers were presented to the didactical framework and in pairs of two, one mathematics teacher and one biology teacher; they designed and implemented interdisciplinary mathematicsbiology teaching sequences. The teachers' reports on their development and implementation of the teaching sequences and presentations given at a final seminar show that in general it is possible for the teachers from two discplines to plan, carry through, evaluate and report about interdisciplinary modeling activities.

Linking Teaching in Mathematics and the Subjects of Natural Science natural science; physics, chemistry and biology. This mathematics and the subjects of natural science based on a discussion of pedagogical and didactical problems concerning the interplay between mathematics and the disciplines of natural science. To exemplify the potential of the framework we then present a case study of an intensive in-service teacher-training program, where mathematics and biology teachers with focus on interdisciplinary modeling activities developed and implemented interdisciplinary teaching sequences.

2 II.

A Didactical Framework for Interdiciplinary Teaching

In International handbook of science education, Berlin and White (1998) argued that science and mathematics are naturally and logically related in the real world, and that educators therefore must try to capture this relationship in the classroom in an effort to improve students' achievement and attitude in both disciplines. The idea of integrated science and mathematics is not new. For example, a historical analysis of documents related to integrated science and mathematics reported by Berlin and Lee (2005) spans from 1901 to 2001. This analysis documents a strong philosophical support for the integration of science and mathematics education as a way to improve student understanding of the two disciplines. It is emphasized that although each of the human enterprises of mathematics and science has a character and history of its own, each of the disciplines depends on and reinforces the other. However, although a great number of research studies have focused on the development of activities and learning materials, following an interdisciplinary approach there is still no emerging framework supporting an integrated mathematics and science education.

Before discussing the problem of a framework for interdisciplinary teaching, we shortly address the notion interdisciplinary without going into a deeper discussion of this multifaceted notion. Repko (2014) stated that interdisciplinary studies is a cognitive process by which individuals or groups draw on disciplinary perspectives and integrate their insights and modes of thinking to advance their understanding of a complex problem with the goal of applying the understanding to a real world problem (p. 28). Obviously, the relationship of disciplinary and interdisciplinary has a productive tension. Interdisciplinary is the study of a complex realworld problem from a perspective of two or more disciplines by drawing on their insights and integrating them to construct a more comprehensive understanding of the problem. With this tension in mind, we apply the term coincident didactic conceptions introduced by Dahland (1998) to express that among the didactics of various disciplines, one can trace a number of analogous notions, which make up a didactic intersection. For example, the didactics of mathematics and biology all include discipline-specific elements, but in addition didactic notions can belong to more than just one subject, i.e. one may talk about intersections of didactic notions. The actual content of such intersections ultimately depends on the perspective adopted. We apply the term coincident didactic conceptions to identify and justify a possible didactical framework for interdisciplinary teaching between mathematics and the subjects of natural sciences. The framework is designed upon three pillars, each addressing a didactic intersection of mathematics and the disciplines of natural science: (i) expansion of domain addressing the applications of mathematical concepts in the disciplines of natural science, (ii) conceptualization of the notion of modeling as an interdisciplinary competence, and (iii) the application of the notion of horizontal linking and vertical structuring to facilitate a path from concrete activities in an interdisciplinary context to conceptual anchoring in the involved disciplines.

3 a) Expansion of Domain

In science education, it is often accentuated that many phenomena and their patterns of interaction are best described in the language of mathematics, which then becomes a bridge between the students' verbal language and the scientific meaning we seek to express (Osborne 2002). However, the description is not straightforward to the students. Firstly, there are differences in terminology and notational systems between mathematics and the disciplines of natural science. The same mathematical structure, e.g. a graph, may apply to different phenomena in a discipline of natural science and hence, the semantics of equal constructs may be very different. Secondly, in the dominating mono-disciplinary teaching approach the teachers presume it obvious that the basis mathematical facts must be apprehended before application in a discipline of natural science. This leads to the problem of transfer, which is one of the biggest challenges in education. It is well known that it is difficult for the students to apply concepts, ideas and procedures learned in one subject, e.g. mathematics, in a new and unanticipated situation, e.g. in biology. Niss (1999) identified the key role of domain specifity as a significant example of the major findings of research in mathematics education. A student's conception of a mathematical concept is determined by the set of specific domains in which that concept has been introduced for the student. When a concept is introduced in a narrow mathematical domain, the student may see it as a formal object with arbitrary rules. This results in the recognized difficulty of application of the concept in new settings. As an alternative we introduce the notion of expansion of domain and point at that interdisciplinary activities between mathematics and disciplines of natural science offer a great variety of domain relations and context settings that can serve as a basis for developing a more practical and coherent structure of a mathematical concept. By expansion of domain to include contexts from the disciplines of natural science, the problem of domain specifity is transcended and the curriculum is presented as a cohesive program (Michelsen 2006).

The notion of expansion of domain aligns with research providing insights into strategies that students might apply to recognize similarities across contexts. Lobato (2003) addressed this central educational issue and argued for a more nuanced and differentiated view of levels of transfer, the actor-oriented transfer perspective. In this perspective, focus is on the type of conceptions that students could have developed given the instructional treatment, and one assumes that learners are making connections between situations nearly all the time, guided by aspects of the situation that they find personally salient. Rebello et al. (2005) considered transfer as the dynamic creation of associations between information read-out by the student in a new situation and a student's prior knowledge. In this kind of transfer, vertical transfer, the student recognizes features of the situation that intuitively activate elements of prior knowledge. The student typically does not have a preconceived knowledge structure that aligns with the problem information. Rather, the student constructs a mental model in situ through successive activation and suppression of associations between knowledge elements. Consequently, the critical issue is to design an instructional environment that supports the students' construction of personal relations of similarities across situations. This calls for an exploitation of meaningful starting points and activities from which conceptual structures in mathematics and the disciplines of natural science can emerge.

4 b) Modeling -an interdisciplinary competence

Models are important in the development of scientific knowledge as they link theories with phenomena. Students' development of potent models should be regarded among the most significant goals of mathematics and science education. The pedagogical power of models comes not just from students using existing models, but also from enabling students to design, build, and assess models of their own (Brady et al. 2015;Gilbert 2004). An extensive research literature recognizes the importance of models and modeling, both in mathematics education and in science education (Halloun & Hesteness, 1987;Gilbert, 2004 Freudenthal (1991) emphasized phenomenological exploration, and argued for that the starting point for mathematics education is those phenomena that beg to be organized. Modelling by mathematization treats specifically the role of mathematics in the disciplines of natural science, and of the link with mathematics in various fields of science education. Pointing at the dramatically change in the nature of problem solving activities and at the difficulties to recruit students capable of graduate level in interdisciplinary such as mathematical biology and bioinformatics Lesh & Sriraman (2005) suggested a bottom up solution. That is, initiate and study the modeling of complex systems that occur in real life situations from the early grades. One could add that the disciplines of natural science offers complex systems to be modeled. This indicates that modeling might provide a generic methodology that can serve as a common ground for learning disciplines such as mathematics, physics, chemistry and biology. Modeling activities take place in an interdisciplinary context and are therefore a possible frame for elucidation of the relations between mathematics and the subjects of natural science.

In Denmark, the notion of subject competences functions as a flexible framework for a description of what is means to master a discipline independent of specific topics and specific levels. Competency is someone's insightful readiness to act appropriately in situations in a way that is guided by one's knowledge from a discipline. Competence-based teaching permeates the Danish educational system, and there are fundamental potentials in terms of an overlap between disciplines. Eight mathematical competences and four science competences are identified, and the competency of modeling is identified both as a mathematical and science competence. The modelling competence in mathematics includes structuring an intra-or extra-mathematical situation to be modelled, mathematizing the situation, analyzing and tackling the model, interpreting the results, validation of the model, communicating about the model, monitoring the modelling activity. The reference to the modelling of extra-mathematical situation underscores that the competence is not specific to mathematics, and therefor modeling should be considered as an interdisciplinary competence. Modeling is a specific problem solving strategy with scientific and pragmatic purposes and as a rule, scientific and everyday life problems call on modelling and do not accept traditional and historical determined boundaries between subjects.

5 c) Horizontal Linking and Vertical Structuring

Historically, mathematical understandings have arisen from nonmathematical preoccupations in the world where increasing refinements of material entities eventually led to the development of ideal objects typical of mathematics (Davis & Hersh 1980, Kitcher 1985, Lützen 2011). There exist frameworks for learning mathematics reflecting this. The notion of emergent model suggested by Gravemeijer (1997) has as the departing point situation specific problems, which are subsequently modeled. The problems first offer the

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Linking Teaching in Mathematics and the Subjects of Natural Sciences by Modeling opportunity to develop situation-specific methods and symbolizations. Then the methods and symbols are modeled from a mathematical perspective and in this sense, mathematical models emerge from the learning activities. The models first come into being as a model of the situation, and then the model gradually becomes an entity in its own right and begins to serve as a model for mathematical reasoning. The shift presented from a model of to a model for should concur with a shift in the way the students perceive and think about the models; from models that derive their meaning from the context situation modeled to thinking about the mathematical content of the models (Doorman & Gravemeijer 2009). Michelsen (2006) suggested an extension of the notion of emerging modeling to include interdisciplinary activities between mathematics and subjects of natural science. The extension consists of two phases: (i) horizontal linking, and (ii) vertical structuring. In the phase of horizontal linking thematic integration is used to connect concept and process skills of mathematics and one or more disciplines of natural science by modeling activities in an interdisciplinary context, e.g. modeling the process of consumption and removal of alcohol. The vertical structuring phase is characterized by a conceptual anchoring of the concepts, e.g. metabolism and concentration in biology and linear growth models, parameters and variables in mathematics, and process skills from the horizontal linking phase by creating languages and symbol systems that allow the students to move about logically and analytically within mathematics and the relevant disciplines(s), e.g. biology, of natural sciences without reference back into the horizontal linking phase. The shift from the horizontal linking to the vertical structuring phase might thus concur with a shift from interdisciplinary teaching to discipline-oriented teaching reflecting the productive tension between interdisciplinary and disciplinary. It should be stressed that the model is iterative. Once the concepts and skills are conceptually anchored in the respective disciplines, they can evolve in a new interdisciplinary context, as part of a horizontal linkage. Thus, the underlying assumption is that the disciplines are themselves the necessary precondition for and foundation for the interdisciplinary enterprise.

To support a learning path from the horizontal linking phase to the vertical structuring phase modeleliciting activities are included. Model-eliciting activities are open-ended, interdisciplinary problem-solving activities that are meant to reveal students' thinking about the concepts embedded in these activities. To get instructional value out of the model-eliciting activities a standard organizational scheme is applied. The scheme consists of a sequence of four phases: (i) warm up activities given the day before to start up the modeleliciting activity, (ii) model-eliciting activities aiming at encouraging the students to work in teams and to express their ways of thinking visible for teachers (iii) model-exploring activities with the goal for the students to develop a powerful representation system for making sense of the targeted conceptual system, and (iv) model-adaption activities with focus on applying the conceptual tool that was developed in the modeleliciting activity and refined in the model-exploring activities (Lesh & Doerr 2003). We notice that the modeling-eliciting activities involve shifting back and forth between among a variety of relevant representations, graphs, tables, equations etc., and the competence of representation comes into play. This competence has an exploratory aspect as the students have to understand and utilize different representations and a productive aspect as well, where the students choose and translate between a variety representations. E.g., in the case of mathematical modeling of a biological phenomenon, the shifts between representations take place in an interdisciplinary mathematics-biology context and draws on biological as well as mathematical knowledge and skills. Consequently, the competence of representation should
The Danish upper secondary education is organized in specialized so-called study packages containing compulsory

disciplines, core disciplines, and elective disciplines. An important feature of a package is that the core disciplines form a coherent program, which is ensured by a closer interaction between the disciplines. Some of the packages include mathematics and biology as core disciplines. To fulfill the objective of coherence in the study packages interdisciplinary teaching across mathematics and biology is demanded. However, the actual classroom practice in Denmark reflects the situation described in the international literature: as a rule, the connections between mathematics and biology in the classroom are weak (Jungck 1997(Jungck , 2005; Cox et al 2016), and the process of connecting the two disciplines should start with the education of teachers (Michelsen 2010, ?orgo 2010). Connecting mathematics and biology is about change, not for the sake of change but for achieving a more comprehensive understanding of core concepts and skills in the two disciplines. This requires that mathematics and biology teachers are prepared to change their minds about the relations between the two disciplines. Real interdisciplinary teaching requires a professional teaching force empowered with the skills necessary for designing learning experiences that

7 a) Mathematics and Biology

Through the times, mathematics was and is inspired by biological problems and as a result, mathematical concepts were constructed and became central elements of the culture of mathematics. An example is the Fibonacci numbers appearing in the pedigrees of idealized honeybees. This is one of the first examples of a population model resulting in exponential growth, and on top of that with a golden growth rate. The immediate effect of Fibonacci's work was not the study of living organisms, but the Fibonacci sequence continued to delight and thrill mathematicians. The wellknown occurrence of the Fibonacci numbers in flowers, e.g. the spirals in the head of sunflowers, led to description by Segerman (2010) of an interesting coloring of the points of the sunflower spiral, involving a "metric" on the positive integers which counts the number of distinct non-consecutive Fibonacci numbers needed to sum to a given number. Jungck (1997Jungck (, 2005)) pointed at, that the absence of strong curricular ties between biology and mathematics misrepresents contemporary biological research, and the need for more mathematics in biology education and problem-solving based curriculum in biology should therefore be addressed. The last century in the history of mathematics is characterized by the increasing influence of applied mathematics. In such different fields as engineering, economics, biology, and medicine applied mathematics has played, and still plays a more and more important role in new development and breakthroughs (Steen 2005). The foundations of many fields of biology and in particular the new fields are inherently mathematical. The method of mathematical modeling applies very broadly in many biological fields including some like population growth and spread of disease (Cohen 2004). Instead of focusing on how to overcome the challenges of implementing mathematics into biology, Jungck (2011) suggested the development of individual biological models that can be easily adopted and adapted for use in both mathematics and biology classrooms. Models and modelling are also suggested as tools to transcend the obstacles preventing the integration of mathematic, physics, and engineering into the biology curriculum and vice versa (Chiel et al 2010).

8 b) Mathbio in the Study Package

In an attempt to offer upper secondary inservice teachers in mathematics and biology didactical tools to prepare themselves for the practical challenges of interdisciplinary teaching, the Laboratory of Coherent Teaching and Learning at University of Southern Denmark in collaboration with the organization Danish Science Gymnasiums in 2015-16 offered the professional development program 'MathBio in the study package'. The overall aim of the program was to enable teachers to implement interdisciplinary teaching sequences between mathematics and biology in their daily classroom practices. The program involved 30 teachers from eight upper secondary schools in the Region of Southern Denmark. It was the core idea of the program to involve teachers in design, implementation, and evaluation of innovative instructional sequences dealing with a wide range of aspects of mathematics and biology.

The program was organized as an intervention project structured around a combination of four seminars at the university and phases of practice at the participating teachers' schools. The teachers were asked to work in pairs, one mathematics teacher and one biology teacher, and they participated in regularly meetings with mathematics and biology education researchers from the Laboratory of Coherent Teaching and Learning. The fundamental aim of the phases of practice was that the teachers designed and implemented an interdisciplinary mathematics-biology teaching sequence. At the seminars the teachers was introduced to the didactical framework for linking mathematics and the disciplines of natural science, different types of organizing interdisciplinary teaching, inquiry based teaching, examples of interdisciplinary mathematics-biology teaching sequences and presentations by researchers working on the interface between mathematics and biology, e.g. computational biology and reconstruction of body size by statistical methods. At the final seminar, the teachers presented their interdisciplinary mathematics-biology teaching sequences at a poster session. Moreover, at a special session at the seminar three elected groups each gave an oral presentation of their teaching sequences, followed by a discussion among all the participating teachers initiated by prepared questions from two selected teachers. To make the improvements in interdisciplinary mathematics-biology sharable and usable for a larger community of teachers, descriptions of all the developed teaching sequences were subsequently made available at the website of Danish Science Gymnasiums.

During the program, the teachers filled in a predesigned protocol to keep track of the development of the teaching sequences. The protocol included fields for teaching and learning goals, content of the teaching sequence with reference to the curriculum, subject oriented and didactical reflections, requirement for didactical supervision, evaluation of the teaching sequence and other issues relevant for the development of the sequences. The written protocols offer insight into the teachers' experiences with the challenges of interdisciplinary teaching, and understanding and application of the three pillars of the didactical framework. Therefore, the protocols were analyzed in order to get an idea of the kind of interdisciplinary mathematics-biology teaching activities that teachers devised, and the type of reflections and experiences regarding interdisciplinary teaching that the teachers themselves addressed.

9 c) The 13 Interdisciplinary Teaching Sequences

Based on selected excerpts from the teachers' protocol we provide an overview of the 13 teaching sequences developed and implemented by the teachers and a discussion of the teachers' reflections about interdisciplinary teaching.

The themes of the instructional sequences address well-known classical mathematical and biological topics like linear, exponential and logistic growth, differential equations, numerical integration chisquare test, probability, statistics, population biology, cell growth, enzymes, Hardy-Weinberg equilibrium, genetics, and evolution. This is probably a consequence of the fact that there in both subjects is a well-defined curriculum, and that teachers are using textbooks, which reflects this curriculum.

The teachers searched for and identified meaningful starting points and connections between the two disciplines. In the majority of the teaching sequences, the biological experiment functioned as the interdisciplinary context connecting biology and mathematics. The experiment delivered data, which were processed by mathematical methods.

Focus is on modeling biological processes by differential equations and the interplay between experiment and theory. (Excerpt from a teacher protocol)

The overall objective of the sequence is that students achieve knowledge of evolution and skills in applying probability theory to model simple stochastic processes. Biology takes the advantage of that evolution randomness principle addressed extensively in mathematics provide can form a solid basis for a focus on the consequences in biology. The mathematical model construction is easily accessible as it is based on an animation and difference equations. (Excerpt from a teacher protocol)

The challenges encountered by the teachers were mainly of practical and technical nature, e.g. the application of different software in the two disciplines.

The biotech experiment provided a better understanding of how to set up and apply mathematical models. The transfer of data from data-collection software loggerpro to math programs is time consuming. (Excerpt from a teacher protocol) Mathematical modeling had a prominent position in the teaching sequences. In the majority of teaching sequences, modeling activities implied a movement from a biological context, e.g. an experiment, to a mathematical context, e.g. and finding the equation of the regression function for the data from an experiment. This is an approach close to what the teachers are used to teach. However, in the teaching sequences developed in the program there are a distinct connection between the learning activities in the two subjects, and therefore a link between mathematics and biology. Furthermore, the teachers were aware of, that the interdisciplinary aspects of modelling open up for addressing concepts like variables, functions and Growth of microorganisms: Exponential and logistic growth, growth rate, serial dilution, microorganisms and fermentation. Brewing beer: Beer, yeast cell growth, factors affecting the growth rate, carbohydrate, biochemistry of metabolism, differential equations, mathematical models, logistic growth, exponential growth, proof technique. Population biology and probability: The Hardy-Weinberg Law and chi-square test.

Population genetics in associated with a study trip to Malta: Sampling of different phenotypic features in Malta and Denmark and comparing the frequency of the two countries.

Cell growth: Virus and pro-and eukaryotic cells' structure and function, linear and exponential growth, modelling of growth, and the limitations and validity of models.

Growth rate and decoloring of Azorubin: Spectrophotometry including the application of the Beer-Lambert law, linear relationships and exponential growth.

Tabel 1: Interdisciplinary mathematics-biology teaching sequences developed by the teachers chi-square test in an interdisciplinary context, and thus transcend the problem transfer.

The students gained a better understanding of the chi-square test. The students benefited greatly from combining the two subjects. The students responded positively to the teaching sequence in the course evaluation. (Excerpt from a teacher protocol)

The goal is to support the students in practicing to apply mathematics in a real situation. Based on experiments made in biotechnology mathematics is applied to plot the data and produce a mathematical model. It is a core idea is the processing of data supports the students' understanding of theoretical math concepts. (Excerpt from a teacher protocol)

The limitations of a model were also addressed in the teaching sequences.

The goal is to apply mathematical models to experimentally biological data and get an understanding of the models' limitations. (Excerpt from a teacher protocol)

Focus is on practical application of mathematical models, especially the models' applicability to biological systems. The students should adopt a critical attitude towards to the model's potential and limitations. (Excerpt from a teacher protocol) Furthermore, the exploratory aspect of interdisciplinary modeling activities involved shifting back and forth among a variety of relevant representations of the concepts involved, which might help the students to ascribe a mathematical as well as a biological meaning to the representations and their mutually relations and by this transcend the language barrier between the two subjects.

The goal is to create an interdisciplinary process, in which mathematics applied in a biological context, to relate observations, model and symbol representations to each other, and to collect process and evaluate data from experiments and taking into account the sources of error, uncertainty and biological variation. (Excerpt from a teacher protocol)

A group of teachers pointed at, that the processing of data might support the students' understanding of variables.

Focus is on modeling the data series with an expected linear relationship between the two methods of counting yeast cells in a liquid medium. The goal is that the students achieve a basic understanding of variables and their relationships. (Excerpt from a teacher protocol) However, a group of teachers drew attention to the disparity between what is considered as good data in mathematics and the data appearing in biology. In mathematics data are "nice" and fit to a well-known functional dependency, and that is not always the case in biology.

The students did not act enthusiastic to cell growth. More attention should have been called to what is considered as good data in mathematics and the kind of data appearing in biology. (Excerpt from a teacher protocol)

In was an issue in several of the protocols, that practical application of mathematics might ascribe a more concrete understanding of an abstract mathematics concept like differential equations.

The overall idea is learn biotech matter based on practical tasks like beer brewing. The teachers and students jointly brew beer and collect data by measuring the mass loss in the fermentation flask. A rather abstract topic like differential equations is applied in a concrete situation. (Excerpt from a teacher protocol)

In some of the teaching sequences a phase with experiments was followed by phase with mathematical as well as biological inquiries. E.g. in the teaching sequences 'Enzyme kinetics 1' the Michaelis-Menten function deduced as a pure mathematical inquiry while the principles of enzyme kinetics were addressed in a pure biological context:

Biology provides data for analysis in mathematics, and mathematics provides an understanding of data analysis in biology. The teaching sequence takes a practical approach to the chi-square test including different games with dice, candy etc. (..). Mathematics offered biology the opportunity to gain a deeper understanding of the Michaelis-Menten function, and biology offered mathematics the chance to work with an equation illustrating the strength of modeling. The students responded positive to the interdisciplinary activities. (Excerpt from a teacher protocol)

The starting point is that the two disciplines are mutually supportive and collaborate in that the biology delivers data, which are processed in mathematics. Mathematics provides an understanding of chi-square test and applies this as a tool in the analysis of biological data. Moreover, mathematics contributes with the deduction of the Hardy-Weinberg law, and offers examples of how we by simulation can illustrate Hardy-Weinberg equilibrium. (Excerpt from a teacher protocol)

The majority of the teaching sequences described a path from an experimental situation in biology to an investigation of a mathematical construct in mathematics, e.g. a graphical representation:

In general, the teachers' reported positive about their experiences with interdisciplinary teaching, and a group of teachers point on the learning potential and A joint mathematics and biology program has positive influence on students' learning. The students developed a subject oriented as well as an interdisciplinary understanding and are well-prepared for future interdisciplinary tasks. (Excerpt from a teacher protocol)

The insight gained by teachers as through participation in the programmed was also addressed:

The sequence was very instructive for us as teachers. It provided us with an insight into the other subject and gave us a glance of the students' activities in another subject. (Excerpt from a teacher protocol)

According to the teachers, the students responded positive to the interdisciplinary approach and experienced a closer connection between the subjects of mathematics and biology. However, it should be noted that some of the students ask for more structure, and that some of the students' focus is on the products of the teaching sequence, and not on the processes linking the two subjects.

We need to be more focused on coordinating and adjusting the process along the way. The students experienced a closer connection between the subjects, but they want a stricter structure with precise and clear requirements for the final product. (Excerpt from a teacher protocol)

The students' evaluation of the sequence was positive. The students appreciated that mathematics apparently is applicable in biology. However, the students' comments show that their focus is on the products of the teaching sequence, and not on the processes linking the two subjects. (Excerpt from a teacher protocol)

IV.

10 Concluding Remarks

The purpose of this paper has been to propose a didactical framework for scaffolding teachers endeavor for realizing widespread desire for a change towards interdisciplinary teaching. As pointed at by Roth (2010) we need to think interdisciplinary from before disciplinary. With the proposed didactical framework we argue that considering modeling as an interdisciplinary competence we can think interdisciplinary from before disciplinary. The excerpts from the teachers' protocols shows that the teachers adapted the three pillars of the didactical framework (i) expansion of domain, (ii) modeling as an interdisciplinary competence, and (iii) horizontal linking and vertical structuring. The didactical framework provides the teachers with a structure for identifying interdisciplinary topics with a significant content for the participating subjects, and modeling serves as the unifying activity in the students' modules.

Looking at the topics of the teaching sequences it is not unfair to say that they are ones belonging to the traditional content of mathematics and biology. This is of course due to a still very discipline oriented curricula, and the fact that the teachers have had their academic training within one or two mono-disciplinary programs. It worth noticing, that a group of teachers addressed the benefit of getting insight into the other subject through participation in the program. And one could for a moment think what it might be, as if learning materials emphasizing the interdisciplinary nature of mathematics and biology were available. Clearly, there is a demand for up-to-date interdisciplinary learning materials to achieve a more integrated curriculum.

The teachers were aware of the potential of expanding the domain of an abstract concept to an interdisciplinary context of mathematics and biology.

The teachers focused on the potential of the interdisciplinary teaching to develop common understanding and language across the two disciplines. This should be contrasted to the traditional approach, where the students are expected to transfer mathematical concepts to a biological context by themselves. With reference to research (Schoenfeld & Arcavi 1988, White & Mitchelmore 1996) showing that a major source of students' difficulties in applying functions is an undeveloped concept of variable, it should be noted that in the teaching sequences variables represent quantities that change and not as only as symbols to be manipulated, and functions are the tool to study the relationships among the changing quantities.

The dominating role of modeling activities in the teaching sequences might be seen as an indication of, that the teachers considered modeling as a competence, which can be applied in an interdisciplinary context and act as a kind of glue between the two disciplines. Evidently, the teachers considered the biological experiment as the obvious common ground for the interplay between mathematics and biology. Exploring the experiment takes the students to mathematical modeling, and they take ownership of their model Concerning the intended path from the in interdisciplinary concrete situation in horizontal linking phase to the conceptual anchoring in the disciplines in the vertical structuring phase, the excerpts from the teachers' protocols show that the path is only unfolded fully in some the teaching sequences. In the majority of the sequences the path is typically from an interdisciplinary context to mathematics, and not to biology too as intended. Firstly, it should be acknowledged that the teachers strive to establish connections between the two disciplines. Secondly, the teachers' experiences with interdisciplinary teaching were limited, and the program was their first encounter with the horizontal linking and vertical structuring approach. Thirdly, the fact that are examples of a path from an interdisciplinary context to the two disciplines indicates that some the teachers adapted the three transfer value to future interdisciplinary activities: The students' positive responses to the interdisciplinary teaching sequences indicates that interdisciplinary modelling activities may motivate the students' learning process and help them to establish cognitive roots for the construction of important mathematical and biological concepts. This is in keeping with research findings showing that interdisciplinary modeling activities contribute to the learning of concepts in the involved disciplines and improve the students' interest in mathematics and the subjects of natural science (English 2013, Michelsen & Sriraman 2009).

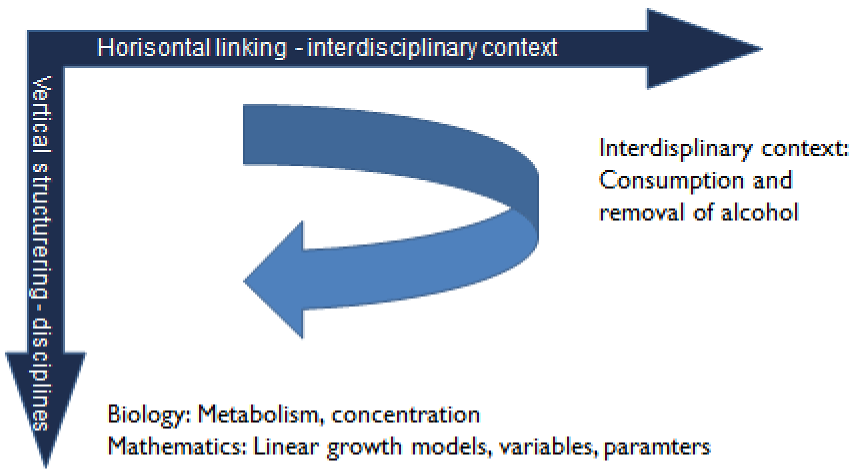
The experiences from the professional development program "MathBio in the study package" in the form of the teachers' reports on their development and implementation of the instructional sequences and the presentations given at the final seminar show that in general it is possible for the teachers from two disciplines to plan, carry through, evaluate and report about interdisciplinary modeling activities. The teachers gained insights regarding their teaching, in particular the limitations of the disciplinary approach and potential of interdisciplinary teaching. Across their disciplines the teachers supported each other in the development and the implementation of the mathematics-biology teaching sequences. The program structure with practice at school, workshops at the university and the final seminar with presentation of the teaching sequences made it possible for the teachers to share their ideas and experiences with their colleagues and having contacts with academic experts in the fields of modeling and educational research. However, to get full profit of interdisciplinary modeling activities further research on the constraints and possibilities of the cooperation between the subjects of mathematics and biology is needed.

The problem addressed in this paper is twofold. On one hand mathematics evidently has played and will play an exceptionally important role in the development of biology, but this role is underrepresented in biology curricula. On the other hand mathematics learned at upper secondary level seems to have little relevance to the biology taught. A primary motivation for introducing interdisciplinary mathematics-biology teaching into the classrooms is the rapidly changing nature of these disciplines as they are practiced in the professional world. Current upper secondary curricula in mathematics and biology don't reflect the interdisciplinary flavor of modern biology, e.g. bioinformatics, and behind the needs of life.

One of the great challenges in the contemporary work of mathematics and biology education researchers and teachers of mathematics and biology is how the interdisciplinary perspective should be reflected in the classrooms. If reform of mathematics education with closer links to biology education is the aim, then prototypes of instructional sequences with learning materials that are in harmony with new perspectives must be adapted by the teachers. A reasonable, yet not exhaustive, answer to this is an increased focus on modeling activities in the daily teaching practice of mathematics and biology. In order to accomplish this, it is important that both pre-service and in-service teachers gain experiences with modelling activities linking mathematics and biology.

The major challenge is capacity building, which is providing support for teachers so that they can develop understandings and skills required to teach for interdisciplinary mathematics and biology curriculum. To work together, mathematics and biology teachers do not need to be experts on each other's subject, but they do need to have a good understanding of how mathematics and biology interact in educational settings. Teachers reflecting about practice through collaboration with trusted colleagues makes the tacit explicit and develops knowledge, skills and expertise in practice. A factor relevant to successful innovations is the degree to which it is perceived better than the existing program it hopes to supersede. Lesh & Sriraman (2005) introduced the main law survival of the useful law that determines the continuing existence of innovative programs and curriculum materials. Usefulness involves going beyond being powerful in a specific situation and for a specific purposes to also be sharable with other people and re-usuable in other situations. It is therefore of great importance to make the improvements available to a larger community of teachers. In 2016-17 a new group of 70 teachers from 19 Danish schools are involved in the second version of the professional development program "MathBio in the study package", and they draw on the experiences from the first version of the program.

V. [1]



1

Figure 1: Figure 1 :

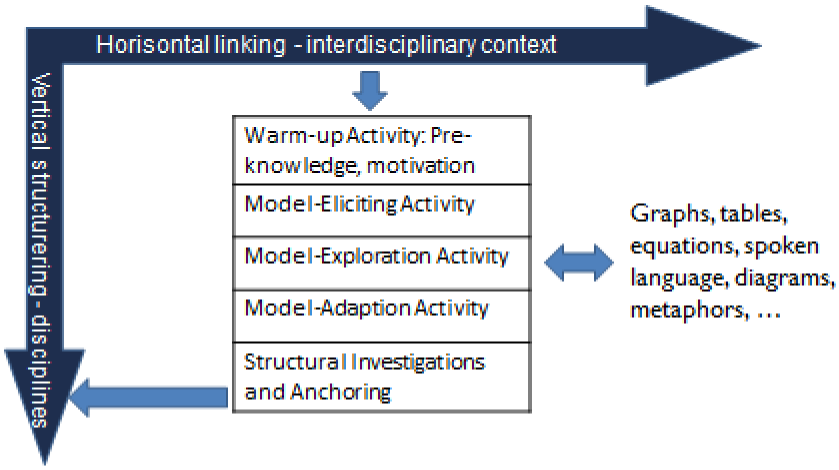

Horisontal linking - interdisciplinary context
Vertical structurering - disciplines
Warm-up Activity: Pre-knowledge, motivation
Model-Eliciting Activity
Model-Exploration Activity
Model-Adaption Activity
Structural Investigations and Anchoring
Graphs, tables, equations, spoken language, diagrams, metaphors, ...

Figure 2:

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