

## 1 Peculiar Features of Verbal Formulations in School Mathematics

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6 **Abstract**7 The article is an exposure to semiotic-symbolic peculiarities of object-based texts which are  
8 the definitions in school math, formulations (descriptions) of some facts and operational  
9 methods which are under study in secondary schools.10 

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11 *Index terms*— semiotic-symbolic peculiarities, object-based texts, definitions, formulations.12 **1 Introduction**13 o exteriorize the contents of school mathematics various semiotic and symbolic means are used [11], [12]. They  
14 allow of fixing -in the expanded or minimized form -the gist of separate objects of assimilation: certain concepts,  
15 mathematical facts (axioms, theorems, formulas, etc.), modes of activity (rules, algorithms, method of solving  
16 problems and proving mathematical statements) as well as their integrity -a fragment of a systematic scientific  
17 theory.18 The text (from the Latin textus -tissue, texture, and web) is the most important way of the expanded reification  
19 of mathematical content formed by means of natural language. Being reflected in the text the essence of concept,  
20 mathematical fact or mode of activity as well as scientific knowledge about their system acquires semiotic-symbolic  
21 reality of life. It is the shell of the text through which the content analysis of scientific and academic noesis is  
22 made. Textbooks, manuals, tutorials and other media represent texts in their visual modality. In their aural  
23 modality the texts are delivered by the teacher in the classroom or by the speaker in audio, visual, and virtual  
24 modes of learning.25 Since the concept of "text" covers two different aspects of the language fixation of mathematical content -at  
26 the level of individual object assimilation and at the level of system of knowledge, one should distinguish between  
27 these cases by entering two different terms into circulation. In the first case, when the text reflects the essence  
28 of a particular object of assimilation, it is appropriate to use the term "objectbased (oriented) text". In the  
29 second case, when the text is a discrete part of a scientific theory, namely -a theme from the school course of  
30 mathematics, it is appropriate to use the term "instructional text". It is clear that an educational text may  
31 contain one or more object-based texts as its components.32 **2 II.**33 **3 Types of Object-Based Texts**

34 In the school course of mathematics the objectbased texts are:

35 -For concepts -formulations of various types of definitions and concept descriptions which are a verbal fixation  
36 of the results (or the progress and results) of certain concept disclosure techniques application; -For math facts  
37 -formulation of axioms, theorems, properties, characteristics and descriptions (verbal equivalents) formulas,  
38 correlations, etc; -For mathematical operations -the formulation of rules, algorithms, heuristic schemes and  
39 unstructured descriptions of the methods of proving mathematical statements, the ways of mathematical problems  
40 solving and more.

### 4 III.

#### 5 The Specificity of the Wording

The logical structure of the formulation depends on the type and characteristics of the object of learning, the essence of which it captures. In its turn, the logical structure of the formulation defines the stylistic design of the corresponding object-based text.

#### 6 a) The formulations of definitions

We consider it right to differentiate between strict and lax formulations. A strict formulation is a logically structured and stylistically perfect text that is constructed according to certain rules of logics and natural language. A strict formulation is devoid of redundancies and expressive content, it is concise and meaningful, its every word being an important text component. Stylistic modifications are allowed, but limited in number. A lax formulation (a correct wording made by the pupils in their own words) can be not logically structured and stylistically imperfect. In the instructional process such kind of formulation has certain didactic functions, both -at the stage of the assimilation object introduction and at the other stages of its mastery by the students. In our opinion, didactically balanced use of lax wording along with the strict wording should be institutionalized in school practice.

#### 7 i. The structure of the strict wording of the definition

General logical structure of a strict formulation of the concept definition reflects the signified concept, a generic term, species differences, and the relationship between them [7]. It can be represented by the following scheme: the signified concept ? generic term ? species differences.

However, the text which is the wording of the definition in its certain stylistic modification can be built in different ways, by inductive as well as deductive principle. In the first case, the text reflects the verbal passage from the particular to the general, and in the second case -vice versa. For example, in textbooks, manuals, tutorials on the methods of teaching mathematics, and reference books [1], [3], [9] the following definitions of the term "Rhombus" can be found:

"A Rhombus is a Parallelogram in which all sides are equal"; "A Rhombus is a Parallelogram with four equal sides, like a square that's leaning"; "A Parallelogram, in which all sides are equal, is called a Rhombus"; "If all sides of a Parallelogram are equal, a Parallelogram is called a Rhombus".

The first two formulations of the definition of a Rhombus directly reflect the logical structure of a general definition of the concept, thus reflecting inductive reasoning -from the signified concept to the generic concept. Therefore, they should be called inductive formulations of concepts definitions. The third and the fourth formulations of the rhombus definition also meet the general logical structure of the concept definition, but reflect a different mental progress, which is fundamentally different from the previous one. Here deductive reasoning follows the principle -from the generic concept, the content of which is separated by some specific features of the signified concept, to the signified concept as a subspecies of the generic concept. This suggests that general logical structure is indirectly reproduced in such definition and relevant texts may be called deductive formulations of concepts definitions.

#### 8 iii. Content-different definitions

It is not only semiotically different but contentdifferent definitions of the same concept which are used in school mathematics. If, for example, the concept of rhombus is determined through a generic term "rectangle" and the related specific properties, then we will have a semantically different definition. It is clear that semiotic-symbolic components of content-different definitions can differ, although the relevant objectoriented text can be built the same way -by inductive or deductive principle.

Thus, different notions of school math can be exteriorized with the help of several semiotic-symbolic means which are text-definitions formed by means of natural language. Fluency in these semiotic-symbolic means should be considered as one of the parameters of a formed concept. iv. Methods that replace or supplement the concept defining procedure In cases when the essential features of the concept have not been sufficiently studied or there is no particular need to do it, one of six techniques that complement or replace the definition are used. These include [7, p. 356]: a) pointing at the object, b) explanation -the disclosure of the concept through the etymology of its name, c) description -the disclosure of the nature through examples, d) characteristicsconcentrating on characteristic features, properties, concepts, and e) comparison -disclosure of the concept through its comparison with other concepts e) distinction -disclosure of this concept through the properties of a contradictory notion.

The progress and the results of the use of such methods of definition are fixed in descriptive texts. They are fundamentally different from texts-definitions bothin their structure which is often not clearly marked, and in their contextual and semiotic-symbolic components. Thus, in teaching mathematics descriptive texts appear as specific semiotic-symbolic means.

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## 9 b) Formulation of mathematical facts

In the dictionary of logical terms [7] the notion of "fact" (from the Latin faktum -done, what is done) is defined as real, actually existing, non-fictional phenomenon or event, something what actually happened, theoretical basis for generalization and conclusion. Mathematical facts that are taught in high school mathematics include axioms, theorems, and formulas. School mathematics is not a strict deductive theory, although it also has content elements which are constructed deductively and the idea of axiomatic construction of mathematics is supported in it.

Those classes of math facts, which are the theorems in deductive theory, in the school course are divided into at least three groups.

The key facts that are most important for the deployment of the logic of the course and teaching high

## 10 ii. Content identical definitions

In the definition of a rhombus the contents of the texts match as the rhombus is defined through the same generic shape -a parallelogram and they employ the same specific properties -the equality of all sides of a parallelogram. However, semiotic-symbolic components of these object-oriented texts are different -the wordings of the above definitions do not match the form of text construction. We can say that in this case the inductive and deductive formulations of the concept definition are identical in their content but semiotically different. Hence, they are to be considered to be various semiotic-symbolic means of the concept essence reification.

For school students mathematical operations are established as a result of the proof. These facts are usually called theorems, although it is not necessarily that this term is used to designate them. For example, the signs of divisibility are not called theorems.

Some basic math facts are introduced into school math without proof. In this case, the students are said that the proofs of such facts exist as they are, but their consideration for one reason or another is postponed. For example, in school basic algebra properties of functions are introduced without proof, and in the geometry course almost all formulas of the volume of geometric bodies are introduced without proof.

Students can encounter some properties of mathematical objects and their attributes in the process of solving problems. These auxiliary facts are usually provided a situational meaning. Students are not required to memorize them. Basic math facts are usually placed in the theoretical part of the tutorial. For students it is them which are the main targets of assimilation.

## 11 i. Common and different features in the definitions of concepts and formulations of mathematical facts

Object-based text containing formulations of axioms, theorems, properties, characteristics, and so on have both common and different concepts with text definitions. Common properties are displayed at the component level -the formulations of mathematical facts are also contextual and semiotic-symbolic components that are subject to certain logical structure. A semiotic-symbolic component -the possibility to exteriorize the meaning of mathematical facts through various text shells -is common.

The features which are different are generated not only through a diversity of objects that describe the formulation of definitions of concepts and formulation of mathematical facts. From the standpoint of semiotic approach the most significant difference we see is that the text-formulations of mathematical facts may reflect some of the information about the object both openly and covertly.

ii. Peculiarities of covert information in the formulation of a mathematical fact. In the formulation of mathematical facts (axioms, theorems, formulas, etc.) covert information can be of three kinds.

## 12 Covert information of the first type

Covert information of the first type is present in each formulation of mathematical facts taught in school mathematics. It is associated with an indirect reflection of the so-called explanatory mathematical fact. The illuminative part of the theorem is a set of objects from which the subset, the elements of which have to do with the conclusion of the theorem, is differentiated.

In each theorem that defines equality or similarity of triangles an illuminative part is a set of pairs of triangles, rather than any other their number. In this set the focus is made on a subset of such pairs of triangles in which the sides and corners are in some relations which are specified in the theorem. Thus, according to the third premise of the signs of equality of triangles in this subset each pair of triangles must have respectively equal sides. It is for these pairs of triangles the elements of this subset, that the conclusion of the theorem in which the relations of equality are made is established.

## 13 Covert information of the second type

Information of the second kind becomes covert when a mathematical formulation of the fact is categorical (affirmative) in its construction. It's the matter of common knowledge that each mathematical fact contains premise and conclusion. They are related to each other either by the relation of implication "If A?, then B?" or

152 by the relation of equivalence "If A?, then B? and if B?then A?" when the mathematical fact maintains certain  
153 criteria (necessary and sufficient conditions) [3], [10].

154 The text membrane of the fact which has an expanded linear structure has a form of an implication. Typically,  
155 it contains the words "if", "then" which serve as specific signs and specific punctuation marks allowing of  
156 separating the text of the premise from the text of the conclusion of theorem. Moreover, the text of the premise  
157 precedes the text of the conclusion. For example, in textbooks every sign-theorem of equality or similarity of  
158 triangles has this type of formulation. The text membrane of the fact which has an unexpanded nonlinear  
159 structure also has a form of an implication. But in this case the text of the premise follows the text of the  
160 conclusion and concerning the two words "if" and "then" the first one is usually present and it fully performs the  
161 functions of them both. An example of this type of text is the formulation of the third sign of the equality of  
162 triangles: "The triangles are equal if their corresponding sides are equal". A categorical (confirming) form of the  
163 formulation can be called a textual semi-expanded shell of the mathematical fact. It does not contain sign-words  
164 "if", "then", the conclusion of the fact has an expanded form and the premise has a minimized form as a rule.

165 Here lies veiled information of the second type. Its recognition largely depends on what type of the verbal  
166 structure of the fact -linear or nonlinear, is realized in the text.

167 For example, a categorical formulation of the theorem about vertical angles "Vertical angles are equal" is the  
168 text of the linear type where the premise of the theorem precedes the conclusion. However, the identification  
169 of covert information, particularly about where exactly the premise of this theorem lies is quite difficult for the  
170 students.

171 Volume XIV Issue III Version I 63 ( ) It turns out that most of the theorems the categorical formulations  
172 of which are based on a linear type, present more difficulties for the students when deploying the premise than  
173 those theorems, the text shells of which have a non-linear type [12]. The fact is that the linguistic peculiarities  
174 of the nonlinear type object-based text mostly require the use of reverse designs, otherwise, most categorical  
175 formulations of nonlinear type will transform into inverse formulations. Additionally, the use of inverse structures  
176 in the formulations leads to a particular semantic distinction of premise and conclusion of the mathematical fact  
177 which facilitates their detection.

178 For example, the affirmative formulation of the divisibility properties of number 10 can be built as a text of a  
179 nonlinear type. In particular, it can acquire the following formulation: "The number with "0" as the last digit in  
180 its record can be divided into 10", "The number with the record ending in "0" can be divided into 10". There  
181 is hidden information of the second type in both formulations. It's much harder to remove it from the first text  
182 than from the second.

## 183 14 Covert information of the third type

184 The fact that the information is covert is permeated by a categorical (affirmative) structure of the formulation  
185 of mathematical facts. We associate its essence with the convolute meanings which appear in interpreting the  
186 terminology used in the text. For example, in the premise of the Pythagorean Theorem firstly a subset of right-  
187 angled triangles is distinguished from the set of triangles, and secondly it is implicitly stated that one side of the  
188 triangle has a length which is longer than the lengths of its two other sides. In the conclusion of this theorem  
189 there has been fixed the existence of the dependence among the lengths of the sides of the triangle, but not  
190 among any of its other elements. In addition, the conclusion reveals the formal meaning of the dependence of  
191 the length of the longest side of the triangle on the lengths of its two other sides.

192 Just as texts-definitions, texts-formulations of mathematical facts which have differences in content and  
193 semiotic-symbolic components should be considered to be different semiotic-symbolic means, even when they are  
194 identically built. For example, semiotic-symbolic components of the listed properties of divisibility of a number  
195 by 10, though built on a nonlinear type, differ significantly, thus, these object texts are different semiotic-symbolic  
196 means. A categorical formulation of the Divisibility by 10, which is also a sample of a non-linear text -"Those and  
197 only those numbers ending with a zero are divided by 10", is different in its content from the formulated properties,  
198 and thus it is also a certain semiotic-symbolic means. So, math facts from the school course of mathematics,  
199 as well as the concepts of this course may be openly exteriorized by several semiotic-symbolic means, which are  
200 text formulations formed by means of natural language. The fluency of operating these semioticsymbolic means  
201 should be one of the indicators of the mathematical fact assimilation.

## 202 15 iii. Shells of mathematical formulas

203 It is not only formulas that can be used to fix the expanded language content of mathematical formulas but also  
204 the object-based texts-descriptions. For example, a verbal analogue of the formula of the square of the sum of  
205 two numbers is: "Square of the sum of two numbers is the sum of the squares and twice the product of the given  
206 numbers" can be considered the formulation. However, reorganizing it in a less concise text and putting separate  
207 semantic units of the text in different sentences, we get a text-description of this formula. Here is an example:  
208 "The formula of the square of the sum of two numbers asserts the equality of two expressions. The first expression  
209 reflects the symbolic record of the square of the sum of two numbers. The second expression symbolically reflects  
210 the result of raising the sum of two numbers to a square. The second expression has three terms the square of  
211 the first term, twice the product of the first and second numbers and the square of the second number".

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## 16 c) The structure of the text-description of mathematical facts

Texts-descriptions of mathematical facts also have contextual and semiotic-symbolic components. Understanding of such texts is largely dependent on the structural features of its iconic and symbolic component. In most cases this component comprises two parts—confirming and explanatory ones. The explanations, with the help of which the hidden meanings are ejected and placed the necessary emphasis on, make the text-description of the formula a rather powerful didactic tool. It is clear that the semiotic-symbolic component of the text-description may be realized in different ways with one and the same semantic component. As a result, each object-based text will act as a separate language semiotic-symbolic means.

## 17 d) Formulations of the methods of operations

In a general sense a method of operation is a system of consecutive activities and operations, the implementation of which results in an outcome that meets the aims of the operation and is adequate. Methods are the most fundamental types of operation that come from the knowledge of the most general laws of objective reality and specific patterns of an object, phenomenon, and the process under research [??]. To indicate those types of operations that are more specific and are used for different specific purposes the term "way" is used. For example, in the methodology of teaching mathematics they distinguish: general methods of mathematics (axiomatic or deductive reasoning, equations and inequalities, coordinate, vector, etc.), methods of proving mathematical statements in which the deductive method is used, methods for solving certain class of problems (a method of the variables substitution in solving biquadratics, a method of using an auxiliary element in geometric problems, etc.). All these ways of operations are the objects of assimilation into the school course of mathematics.

In the structure of the ways of operation they differentiate content (epistemological) and operational (activity-based) components. The content component of the way of operation is a system of knowledge which includes: initial knowledge about the object and its properties, the final knowledge of the results of operations with the object, knowledge of the operating mode of activities (actions and operations which in a definite sequence realize the way of operation), knowledge of subject-practical means needed to perform the activity, the guidance system of the choice of some ways of operation among the others. The operating component of the operation mode associated with the direct performance of its actions.

The acquisition of the semantic component of the way of operations by the students is characterized by such new constructions in their personal experience as knowledge, and mastery of the operational component is reflected in skills.

## 18 i. Structure of the operation way formulation

The wording of the rules, algorithms, and heuristic schemes in school mathematics are also contextual and semiotic-symbolic components that are subject to certain logical structure. In general terms, it can be represented as: the object ? operations with the object ? the result.

The deployment of this scheme in the text of formulation can occur both in the linear and nonlinear modes. If the text has a linear structure it describes the actions with the object and then -the obtained result. If the text has a nonlinear structure everything is done vice versa.

Most of the rules, algorithms, and heuristic schemes of school mathematics which are the open objects of learning have a nonlinear structure. For example:

-To divide fractions the dividend should be multiplied by the number inverse of the divisor [14]; -To solve the system of equations through a substitution method, you should: 1) express one variable of any of its equations through the other; 2) substitute this variable by the resulting expression in a different equation of the system; 3) solve a created equation with one variable; 4) find the corresponding meaning of the second variable [1].

ii. Specificity of non-linear texts Object-based texts of the non-linear type are implicative. In these texts the operations with the object are separated from the result these actions will arrive at. The operations and the result are connected by consecutive relations though the cause and the consequence in a nonlinear text are reflected in inverse order. Such texts are devoid of the words "if", "then" with their functions being performed by the phrases: "in order to ... you are to ... ", "to? one should?" and so on. However, every rule of the school mathematics can be formulated in a purely implicative form. For example, the rule of fractions division in this case may look as follows: "If this fraction is multiplied by the number which is inverse to the other number, we get a share of the division of this fraction by this second number".

With the same semantic component, different text shells of the rule, such as the rule of dividing fractions, act as different semiotic-symbolic means.

## 19 iii. Peculiarities of the linear type texts

Linear formulations of operation types are rare in school textbooks. For the most part, formulations of operation types are categorical. The formulations of the rule of multiplying two fractions (mathematics tutorial for the 6<sup>th</sup> This fractions multiplication rule can be interpreted as a mathematical formulation of the appropriate fact. It depends on what meaning -that of the result or the procedure is given to the word "product". In a procedural

270 sense in this text it means the operation of finding the product of two fractions, and hence the first part of the  
271 text, "the product of two fractions is ?" carries the meaning "if we multiply two fractions, the product is obtained  
272 as the result of the following steps ...". It is in this case that the given text will act as the formulation of the  
273 multiplication of fractions. If we understand the word "product" in the resulting sense, then the formulation  
274 becomes affirmative which is a sign of the mathematical fact formulation. form) is the text of this very kind:  
275 "The product of two fractions is a fraction, the numerator of which is the product of the numerator and the  
276 denominator is the product of the denominators" [8].

277 Thus, the same text shell can cover the contents of different objects of assimilation. It is clear that with different  
278 content components, although with the same semiotic-symbolic components of such texts, they represent different  
279 semiotic-symbolic means.

### 280 **20 e) Non-verbalized rules of the application of definitions and** 281 **formulations**

282 In studying mathematical concepts and facts the non-verbalized rules of their use serve as implicit objects of  
283 assimilation as the correct performance of the relevant operations even without them being verbalized is an  
284 indicator that the content of a concept or fact has been mastered by the students.

285 Let's consider the definition of the concept of a standard form monomial provided in the algebra tutorial  
286 [1]. "If the monomial has only one numerical multiplier which is put in the first place and if each variable is  
287 included into only one factor, such monomial is called a standard form monomial". From this definition follows  
288 the following rule of the reduction of the monomials to the standard form:

289 "To reduce the monomial to a standard view, you are: 1) to multiply numerical factors; 2) to place the resulting  
290 number first in the record of standard monomials; 3) to group the multipliers that contain the same variable; 4)  
291 to find the product of similar alphabetic factors in each group using the rule of multiplication of powers; 5) in  
292 the record of a standard monomial to place the derived products of monomials after a numerical factor and in  
293 alphabetical order".

294 In such expanded form the rule of the monomial reduction is not offered to be learnt by school children.  
295 The main object of assimilation is the relevant definition. However, the index of conscious understanding and  
296 mastering of this definition is not only the ability to reproduce the wording of the definition, but the ability  
297 to perform correctly the above mentioned sequence of operations. It does not matter whether the student can  
298 verbalize the meaning of the actions performed. The very fact of the correct performance of operations is of  
299 importance here. Thus, each object-based text containing definitions of the notion and the formulations of the  
300 theorem, properties, attributes, and formulas should be considered in two ways -affirmative and procedural.  
301 Hence, in every object-based text except semantic and semiotic-symbolic components one should see a functional  
302 component that will influence the variation of the corresponding semiotic-symbolic means of external fixation of  
303 mathematical content.

### 304 **21 f) Texts-descriptions of ways of operation**

305 Texts-descriptions used for the expanded content reification of the content of the ways of mathematical operations  
306 differ from the textformulations by their unstructured semiotic-symbolic component. For example, the following  
307 text contains the information about the rule of finding the ratio of two numbers in a non-structured way: "In  
308 mathematics there is a convenient way of comparison of similar quantities, which is that to compare quantities  
309 they seek an answer to the question, how many times one quantity is larger than another. The answer to this  
310 question is found by dividing" [8].

311 For the most part, by means of unstructured texts the content of the methods of proving mathematical  
312 statements, means of solving problems, and more is fixed. Like similar text shells of mathematical facts, they  
313 have two parts -confirming and explanatory ones. The texts of the descriptions will vary depending on how their  
314 semiotic-symbolic components are built: whether a confirming part is represented explicitly, how transparently  
315 the explanatory part explicates the necessary core content, on what basis -inductive or deductive -the text-  
316 description is built. However, any variations will present a new semiotic-symbolic means of reification of the  
317 same method of operation content.

318 IV.

## 319 **22 Conclusions**

320 Educational function of teaching mathematics as a major feature of modern school mathematics education lies  
321 in it that students should master a certain amount of social experience and human knowledge which will help  
322 them discover and develop their cognitive and human potentialities, needs, interests, and facilitate their self-  
323 actualization.

324 The content of school mathematics education as an abstraction and its reification by semioticsymbolic means in  
325 different modalities (visual, auditory, and kinesthetic) requires from the students learning school mathematics, to  
326 grasp a several semioticsymbolic systems in full volume and at a certain level. It also demands special knowledge  
327 and skills to transfer one semiotic-symbolic system into another, including the transfer of visual, tangible assets

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328 or plastic substitutes into a verbal system and vice versa. Some semiotic-symbolic system, including some  
329 subsystems of mathematical language should be a means of students' further education and development, and  
330 therefore should be regarded as objects of assimilation in teaching and learning mathematics in school.

## 331 23 Literature



Figure 1:



