Teacher's Conceptions and Beliefs in Orienting the Solution of Problems of Additive Structure

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Abstract

This article describes the concepts and problem types, performed by three primary education teachers around the teaching of mathematics and addition in particular. We applied the quantitative research approach with a design case study section, using a Likert scale questionnaire, a test of personal constructs and a self-report. The results show that in the scheduled classes, the prevailing tendency of traditional teaching and technology. It became apparent dichotomy between what teachers think about mathematics and their teaching. The additive problems are referred to written statement and numerical exercises; whose characteristics correspond to problems in routine phrased containing solution strategy either directly or indirectly.

Index terms— teachers in service, conceptions, problems of additive structure, orientations, problem solving.

Resumen-En este artículo se describen las concepciones y tipos de problema que desarrollan tres profesores de educación básica primaria en torno a la enseñanza de la matemática y, en particular de la suma. Para ello se aplicó el enfoque de investigación cuantitativo, con un diseño de estudio de caso transversal, aplicando un cuestionario de escala tipo Likert, un test sobre constructos personales y un auto reporte. Los resultados muestran que, en las clases planificadas, prevalece la tendencia de enseñanza tradicional y la tecnológica. Se hizo evidente la dicotomía entre lo que el profesor piensa sobre la matemática y su forma de enseñarla. Los problemas aditivos propuestos, son referidos a enunciados escritos y ejercicios numéricos, cuyas características corresponden a problemas de rutina que contienen en su enunciado la estrategia de solución, ya sea directa o indirectamente.

Introduction n order to guide students in solving everyday problems, the mastery of additive structures is essential in a mathematics teacher (Chinnappan and Thomas, 2001), since these include the basic primary operations,which the student must base its background knowledge in mathematics, demanding a great effort from the student to appropriate the concepts that are put into play (Kieran et al., 2016;Radford, 2018).

Bryant, Nunes and Tzekaki (2009) affirm that the first steps of children in mathematical reasoning follow directly from their experiences in additive reasoning, so any type of limitations in the domain of problems of additive structures leads them to commit serious mistakes in solving mathematical tasks. If this is taken into account, unless teachers can really address the problems of additive structures properly, it is unlikely that they can help children move forward in an adequate development of their mathematical thinking (Willis and Fuson, 1988). In this sense, it is necessary for the teacher, to properly master these structures and to be competent, guiding his students towards their understanding, and also to be knowledgeable about adequate learning theories to base their practice, and the implications of these theories that allow students to align with the contents that are oriented to them (Ball, Thames and Phelps, 2008). This demands from the teacher a great preparation to be able to anticipate what their students will do, what they think, which in turn can provide information on how they make sense of the mathematical contents, by connecting their understanding of the operations and procedures that they use to solve the task, with the semantic characteristics of the problems they solve (Chapman, 2007;Dolores, 2013).

Throughout this process, conceptions the teacher has about the way of teaching are very involved, because the conceptions about the teaching of mathematics plays a very important role in the development of teacher training, this is because each teacher may conceive the concepts he teaches differently, so it is possible that each one emphasizes different aspects hoping to find some coherence with his own conceptions (Lebrija, Flores and **??**rejos, 2010, Arcavi, 2020).

Within the context of the described context and consequently assuming the fundamental role that teachers have in the educational setting, a research process was developed, guided by the following question: What are the conceptions of primary school teachers in guiding children solving problems of additive structures?

1 II.

2 Theoretical Approach

To answer the question posed, it was sought to describe the conceptions and types of problems that teachers of basic primary education develop around teaching how to solve additive structure problems.

3 III. Mathematics Teacher's Conceptions

and Beliefs in Practice Gil and Rico (2003), describe beliefs as the undisputed personal truths sustained by each one, derived from experience or fantasy, a strong evaluative and affective component, through which you can understand and characterize the ways they have to interpret teaching and learning. They propose to focus attention on the conceptions and beliefs of math teachers, because knowing them you can better understand some of their attitudes and positions. In addition, consider that each teacher gives a personal answer to the key questions of the curriculum for their action in the classroom: it has some objectives, but to achieve them it works some contents with a certain methodology and applies some evaluation criteria. To Flórez and Solano (2011) and Heuvel-Panhuizen (2020), conceptions appear as another important structure to describe human thought. However, they state that they are difficult to define, thus, beliefs can be seen as incontrovertible personal truths that are idiosyncratic, with much affective value and evaluative components. Likewise, conceptions are considered as "implicit organizers of concepts, of an essentially cognitive nature and that include beliefs, meanings, concepts, propositions, rules, mental images, preferences, etc., that influence what is perceived and processes of reasoning that are carried out" **??**Azcárate and Moreno, 2003, p. 267). In this sense, the Ministry of National Education (1998) suggests teachers reflect on what to teach? when to teach? how to teach? And what, how and when to evaluate? as fundamental elements, pillars of the teaching and learning process.

Regarding the studies carried out on teachers' conceptions and beliefs about the nature of mathematics and the relationship they have with their practice in the classroom, Suárez, Martín and Pájaro (2012) consider their practice to be dialectical, that their beliefs and conceptions affect the teacher's practice, but in turn the practice can cause the teacher to revaluate their beliefs and conceptions.

To Muis (2004), beliefs that affect the decisions teachers make in math class can be classified into three basic types: ? Beliefs about mathematics: on the one hand, there is the aspect of those who believe that mathematics is finished, absolute knowledge, which is constituted by a relation of fixed and infallible concepts, which must be memorized in order to be learned. Another aspect of this belief is that the individual invents or creates mathematical knowledge according to the needs of science or those of everyday life, so that knowledge is constantly and continuously modified. ? Beliefs about how to learn mathematics: these can be located in two extremes: one in the belief that the student plays an active role in the construction of his own knowledge, so the conditions must be provided for them to develop their potential, analyze and defend or refute views on the solution to a problem. On the other hand, the belief that the student is a mere receiver of knowledge, so the strategies used in the instructional processes must be to dictate notes or exercise, following a model previously made by the teacher. ? Beliefs about teaching: this, like the previous one, can also be located at two extremes: one where it is believed that teaching is the center of the knowledge acquisition process, and that in order to acquire it, students must exercise and memorize concepts and procedures. At the other extreme, there is a belief that teaching a student implies leading him to think like mathematicians, and that teaching should be oriented to the understanding of concepts and procedures as a means to solve problems. Likewise, it is believed that it is necessary to adapt the teaching to the characteristics of the knowledge and to the cognitive and affective needs of the students.

Contreras (2010) built a profile of the didactic trends of a math teacher, based on his beliefs about the role of problem solving in the classroom. Discover, little concordance between the conceptions that a teacher has with specific tendency, presenting a diversity of possibilities between the relationship of his conceptions and the simultaneity of tendencies for the same teacher, so there are differences between the tendencies of one teacher to another. Based on his findings, he suggests some trends that can be established according to the different ways of manifesting. It highlights factors involved in the teaching and learning processes, which can affect these beliefs: the methodologies, the purposes of the subjects, the role played by students, teachers and the evaluation carried out in said process. Contreras proposes to work by solving problems as an instrument to produce a change of conceptions about mathematics and its teaching and learning.

In this regard, Hernández (2011) considers that the analysis of student attitudes for mathematics teachers is an issue that has aroused the interest of research in mathematics education, since the inadequacy of traditional approaches to achieve the objectives of an increasingly demanding and changing society. That is, the knowledge that is conceived by the undergraduate is outdated before the teacher leaves, so the need for the qualification to be continuous and permanent is pressing. In addition, this process of permanent outdating of the knowledge acquired, even before using it, suggests that it should be developed are adaptive skills, rather than updated and useful content for specific issues.

To Gamboa (2014) the affective dimension, closely related to beliefs, is a very strong determinant in the learning of mathematics, so this element must be taken into account by researchers in mathematical education as a means to understand this process from the perspective of both students and teachers. He considers that from his study a change in this discipline could be achieved, since everything seems to be a matter of attitude to achieve an improvement of the beliefs and attitudes of students and teachers towards this area of knowledge. Gamboa (2014) states that mathematics is presented in the school curriculum as one of the most feared subjects, which causes students to reject it, which leads to difficulties and low levels of achievement in their teaching and learning process. Despite the above, Hernández (2011) indicates that mathematics have usually been related to rationality, abstraction and logical reasoning, so that their learning must be linked to the formation of positive attitudes, not only by mathematics, but of mathematics as a way to enhance other areas of knowledge. For Hernández (2011), mathematics is a dynamic in students, which functions as a trigger, contributes to logical reasoning when dealing with situations in other sciences and as a conceptual organizer that facilitates interactive regulation between equals.

Regarding the theoretical model corresponding conceptions of mathematics **??**odino, Batanero and Font (2003), affirm that beliefs about the nature of mathematics (idealist-platonic and constructivist) are a factor that determines the performance of teachers in the class. Zapata, Blanco and Contreras (2009) use three trends: platonic, instrumentalist and problem solving. The platonic view considers mathematics as a body of static but unified knowledge, as an immutable product which is discovered, not created. On the other hand, the instrumental vision assumes mathematics as a tool bag, which is composed of an accumulation of facts, rules and skills that the trained craftsman must use skillfully in search of some external purpose. In this way mathematics is a set of useful and separated rules and facts. The problem-solving vision states that there is a dynamic, where mathematics viewed as a field of creation and human invention that is constantly expanding (Heuvel-Panhuizen, 2020), within which patterns are produced and subsequently distilled in the form of knowledge, which is added to the total of knowledge mathematics is not a finished product, as its results remain open for review.

4 IV.

5 Additive Structures

Making an approach to the conceptual dimension of numerical thinking, according to Romero et al. (2002), when talking about additive structures, reference is made to mental conceptions and images which in a constructive process, who learns them gradually builds up, from which they give meanings to situations involving numbers natural, addition and subtraction of numbers, in order to understand them, make sense and find strategies to address them.

According to Bonilla, Sánchez and Guerrero (1999), problems with additive structure are those solved with an addition or subtraction operation. The symbolic problems of additive structure will vary according to the open sentence given in the problem. Changing the unknown generates six open sentences for the sum and another six for the subtraction. The classification of problems that are carried out according to their semantic structure is considered of great interest. Four categories can be considered in school verbal problems that suggest addition and subtraction operations: change, combination, comparison and equalization.

According to Orrantia (2003), exchange problems are made up of an amount to which something is added or removed, resulting in a new amount. The problems of combination and comparison are made up of two quantities that are combined or compared to produce a third quantity. Those related to equalization are composed by a quantity and a result and the missing quantity that leads to that result is requested with an addition or subtraction operation. The first three types of problems reflect the same type of actions to be performed and the last, suggest the use of an equation to find an unknown or operate by trial. However, since the problems include three quantities, one of which is unknown, in each category several types of problems can be identified according to what quantity is unknown. The following table shows the typology of structures that may result when combining additive structure sentences. Some examples of these type of problems in order of complexity and use are:

"Carlos has 4 apples and 5 pears. How many fruits do you have in total? "(C1).

"Before he started playing, Andrés had 8 marbles and won 5. How many marbles does he have now?" (C2). "Julio has 2000 pesos less than José and he has 1500 more than Ana. How much does Ana have more than Julio?" (C6).

V.

6 Methodology a) Type of study

This work was developed under a mixed (Creswell, 2009), where the quantitative component (Hernández, Fernández and Baptista, 2006), corresponds to a non-experimental research design, since there was no manipulation of variables and the phenomenon was the observed object of the study in its natural environment. The qualitative components are the actions and reasons given by teachers in the development of their math classes, related to the resolution of problems of additive structures. A descriptive case design was made (Hernández et al., 2006), and following Mertens (2005) individuals, were seen and analyzed as an entity. In this study, the case was the conceptions that teachers have about teaching and learning of mathematics, in relation to the teaching of problem solving of additive structures.

7 b) Sample

Informants in this study were three teachers from elementary level of education that guide the area of math in third, fourth and fifth grades. The inclusion criteria: at least five years of experience; with residence in the urban area to facilitate contact; availability to participate and attend the research process, and to be entitled as a math teacher. The teachers age was 37 and 45 years old, and all had been working as a math teacher for more than ten years. For the analysis of the information they have been given fictitious names (Sara, Juan and Carlos), to protect their identity.

8 c) Study Variables

The variables observed and analyzed in this study were: 1) Conceptions about the nature of mathematics, teaching and learning of mathematics, teaching and learning of problems of additive structure; 2) Teaching trends or didactic model used by the teacher, and 3) Type of problems of additive structure addressed.

9 d) Information Gathering

To collect the information, four instruments were applied: (1) a Likert scale questionnaire. The information collected in it, allowed a first characterization of the teacher's conceptions, taking into account aspects such as: attitude towards mathematics, vision towards mathematics, attitude towards the teaching of mathematics, vision of the teaching of mathematics, vision of learning mathematics. (2) The technique of actions and reasons, within the technique of the mesh applied by Rodríguez (2003): here, each teacher stated actions and reasons (between 15 and 25) that he normally proposes during the development of his math classes and particularly when developing topics related to the solution of additive structure problems. With the actions and their respective reasons, each teacher completed a square grid or grid, from which a matrix resulted allowing to build a database in the SPSS program. (3)Teachers were asked to plan a lesson which, in a first activity, allowed to gather information focused on the experience of each teacher, for this each teacher was asked to work on the concept of addition in one class and, in another, the subtraction. And (4) each teacher was asked to formulate six situations or activities that required for its solution, the addition or subtraction operations, this the claim to investigate the types of problems used by teachers in the classes. Thus, an approach was made to the conceptions of each teacher from each instrument applied.

Information processing was carried out through statistical methods, seeking to avoid to the maximum that the observed or measured phenomenon was affected by the personal preferences of the researchers. The method of extraction of principal components and rotation analysis (Varimax normalization with Kaiser) was applied, in addition, by means of factorial analysis, groups or clusters of reasons were closely related generated. Each group was assigned a generic label or name that gathered the essence of the reasons that constitute each group or conglomerate. An individual analysis of each case was performed, with all the instruments, then a characterization of each teacher was made, taking into account the aspects proposed for the analysis. In addition, a comparative analysis was made between cases, crossing the results obtained for each one, so that an approach to the shared aspects of the teachers under study was obtained, in relation to their conceptions.

10 VI. Results and Information Analysis

The analysis of the information collected shows that three teachers (Sara, Juan and Carlos) presented a positive attitude towards teaching mathematics. Similar to Lebrija et al. (2010), inclined favorably towards the problem-solving vision, and negatively towards the instrumental and platonic vision, the latter being the least favored. Faced with the vision of the teaching of mathematics, they favorably shared the vision of teaching by discovery, using several solution strategies, with a cooperative learning vision, showing sufficiency in their work, but without visualizing themselves integrated as a team in a collaborative work with peers in their area (Rodríguez and Espinoza, 2017). Likewise, they unfavorably assumed the vision of teaching the textguided curriculum. On the vision of teaching focused on basic skills and the vision of problem solving (Muis, 2004), both Sara and Carlos express a negative attitude. In relation to the vision of teaching from a curriculum designed by the teacher, only Sara and Juan show a positive attitude.

In relation to the vision of learning of mathematics (Gamboa, 2014), teachers unfavorably agree with memory learning. In the constructivist vision of learning and the role of errors in teaching, Juan and Carlos expressed their acceptance, while Sara assumed a negative attitude. When learning from the decision and autonomy, Sara and Juan shared the favorable attitude, while Carlos assumed a negative attitude. Although it is observed that they share some beliefs, the heterogeneity between them is also appreciated, an aspect that leads to the sharing of the position of Gil and Rico (2003) when they express that one cannot speak of a homogeneous and organized knowledge of mathematics teachers about their teaching and learning, since they are influenced by their opinions and personal experiences.

The characterization thrown according to the groups or conglomerates of related reasons found and labeled, are presented in Table 2, where, in addition, the preferences of each teacher are described. For each case, the set of labels assigned for the different groups of ratios obtained by the method of extraction of main components and of rotation (Varimax normalization with Kaiser) is presented. Question as a control factor. The teaching orientation of the teacher to encourage learning. Evaluation and group work as a promoter of responsibility and learning. Emphasis on consolidation and concrete activities. Prior knowledge as a factor for understanding. Situations and problems of the context as factors of integration and connection of concepts. Repetition as a mechanism to strengthen learning.

11 Source: Self elaboration.

Looking at different ideas each assigned label contains (table 2), it can be seen that the three teachers a group of shared reasons prevails when thinking about the design of the class. These actions are aimed at: reinforcing or strengthening the issue, determining prior knowledge, generating interest and motivation, working in groups, evaluating to verify/control and guide/explain the issue. The planning of the classes, are actions shared by the teachers: masterly presentation as usual technique and use of the textbook as the only curricular material, an aspect that seems to follow the structure of a behavioral pedagogical model.

The initial diagnosis they make of their students, is based exclusively on the contents that, supposedly, have been taught previously. These aspects are characteristic of the traditional didactic tendency, which as described by Parra (2005), is based on deductive activities with a methodological structure theoryexample-exercise, which consists of an explanation of the teacher, followed by the presentation of an example, to finally assign a series of exercises where the oriented contents are applied. In this order, the teacher verbally transmits the learning contents, through the dictation of his notes or allusion to a textbook, where the exam is the ideal instrument to measure the students' learning, in addition, the student must dedicate an express time for its preparation.

Being the evaluation one of the most relevant aspects in the training processes, it could be expected that through it, it will realize the development of competencies in those who learn (Tejada and Ruiz, 2016;Scherer, 2020). As evaluation as an integrated element of the educational process, it should be of great impact on students, but if it does not fulfill its formative role it is reduced to measurement for certification (López, 2012). According to Canabal and Margalef (2017) and Contreras-Pérez and Zúñiga-González (2017), for the evaluation to fulfill this formative role, it requires the active presence of feedback, however, in light of the results, this is perceived as deficit (Ion, Silva and García, 2013). Now, from a comparative view to the teachers' plans, as in Zapata et al. (2009), it is appreciated that, within the conceptions of mathematics teaching, the teaching trends that prevail in common are traditional and technological. According to Zapata et al. This predominance of traditional education could be justified by the tendency of teachers to reproduce, especially during the first period of their professional practice, the models in which they have been trained, as if there were an involuntary extension of the actions of their education teachers Basic, medium or university, who survive resiliently for some time in their school practices.

Regarding the analysis of types of situations and problems used in class planning, we agree with Martínez and Gorgorió (2004) that the proposed situations were referred to problems of written statement or numerical exercises, the problems were reasoned or in failing that, numerical operations exercises. Parra (2005) calls it a timid incorporation of problem solving. Likewise, it can be seen that the use of records and representations by these professors in their professional practice is quite restricted (Martínez, 2003).

Data show the conceptions teachers have to work with problems of additive structures at school, which could be called "written narration of a mathematical situation" **??**artínez (2003, p. 260). This apparent absence of problems with a variety of information representation in the math class has, according to Chapman (2007), important didactic consequences, such as limiting the use of representations and their role as a mediation tool in problem resolution.

The groups of factors labeled to characterize each teacher when planning the classes, show some characteristics of a constructivist and sometimes social cognitive work, since they say emphasize teamwork, error monitoring and previous knowledge, as a factor to understanding, as well as the use of context problem situations, as a factor of integration and connection with concepts; However, the proposed activities, the indications given and the way in which they are developed are behavioral.

Regarding the categories (Bonilla et al., 1999;Orrantia, 2003), a high percentage (78%) of the problems proposed by the teachers correspond to problems in which two measures are composed to give rise to a new measure. Also, 89% of the proposed problems are of the structure "a + b =?", Where the unknown quantity is located in the final measure, given the initial measures a and b. An example of this are those presented by Carlos: "I have 5 apples and 3 pears, how many fruits do I have? Or in one hand I have $ 420 pesos and in the other $ 80 pesos. How many weights do I have? These types of problems according to the Ministry of National Education (2010) correspond to routine problems, information that is relevant, because there is still a concern in teachers to present students with the same type of scheme or structure in the problems; avoiding to pose more complex problems where the unknown is not the search for the final measure. None presented problems of equalization, where you had to use the concept of equation to find an unknown quantity.

12 VII.

13 Conclusions

Inquiring about the conceptions and types of problems that primary school teachers develop around the teaching of mathematics and in particular of problems of additive structures, allowed us to conclude that the prevailing conceptions of teaching mathematics are emphasized in traditional and technological trends, which according to Zapata et al. (2009) are unfavorable for the development of thinking processes and skills in mathematics. These conceptions emphasize the role of the teacher and the passivity of the student.

Despite the fact that the three teachers have a high level of acceptance for the vision of mathematics as problem solving, they also present a high level of rejection to the vision of teaching from the resolution of problems, a situation that makes the permanent dichotomy evident. There are some inconsistencies between what the teacher thinks about mathematics and the way they teach it. This situation could be explained from what was stated by Rodríguez (2003), especially as dichotomy and fragmentation.

From the analysis made to the actions and reasons presented by the professors at the time of the design of the class for teaching the solution of additive structure problems, it is appreciated that these actions are directed mainly to: reinforce or strengthen the subject, determine the previous knowledge, generate interest and motivation, work in groups, evaluate to verify and control or to certify, to guide and explain the subject, all this supported by the presentation of exercises that promote mechanization and algorithm.

A low level of coherence was found between each teacher's conceptions and their teaching tendencies. In this particular case, the limited knowledge and training that teachers have around the teaching of mathematics, seem to restrict the possibility of implementing them in teaching practice, encouraging traditional pedagogy that does not encourage the student to think so that in this way can develop thinking skills and be able to develop mathematically competent. Well, they just facilitate memorization processes, disconnected from the socio-cultural context where learning takes place. All this, despite the fact that the vision of teaching they claim to have, is that of problem solving.

The tendency of teachers to introduce students to problems in the simplest structures dominates, where the unknown quantity is the final measure or routine in its statement containing the solution strategy either directly or indirectly. The preference for these types of structures has limited the approach to more complex problems that lead students to make stronger reflections. In addition, the approach that teachers make of situations are referred to problems of written statement and numerical which does not facilitate an approach to the various forms and structures that additive problems may have, also limiting the field of experience students could explore. [1] [2]

Figure 1:

2

Sara		Juan	Carlos
Teaching reinforcement and		Emphasis on evaluation	
strengthening	mechanisms.	and motivation. Verify and	
Emphasis	on previous	reinforce the subject.	
knowledge. Connection and		Attention to doubts and	
relationship	with the	errors. Explanation for the	
environment.	Actions for	appropriation	of the
Monitoring and evaluation of		subject.	
achievements. Emphasis on			
group	work and		
communication. Emphasis on			
motivation.			

Figure 2: Table 2 :

[1](G)

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