

1 Possible Gnomonic Algorithm & Calendar from Vr?ac's Circles,
2 Serbia

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5

6 **Abstract**

7 Vr?ac Circles are a set of five, practically, concentric circles located in a plain of western
8 Serbia. There have not been any systematic archaeological research in the zone but some
9 exploration work indicate a possible date for the earthwork to be around five thousand years
10 ago. In this paper, we present a sequential analysis of the sizes (diameters) of the circles that
11 may obey, with the help of a gnomonic approach (the "gnomonic factor"), an algorithm
12 involving the number of the circle and the height of the gnomon. The first assumption is that
13 the smallest radius would be obtained from the shadow of a gnomon on winter solstice at
14 noon; the others by the addition to the first a multiple of 1.5 times the gnomon's height. At
15 the final stage of the analysis we discuss a possible calendrical meaning of the circles. Our
16 results, we consider, call the archaeologist to look at this place for confirmation or rejection of
17 these hypothesis. They may support subsequent development of ideas for the history of
18 mathematics.

19

20 *Index terms—*

21 **1 Vr?ac's Circles, Serbia**

22 Abstract-Vr?ac Circles are a set of five, practically, concentric circles located in a plain of western Serbia. There
23 have not been any systematic archaeological research in the zone but some exploration work indicate a possible
24 date for the earthwork to be around five thousand years ago. In this paper, we present a sequential analysis of
25 the sizes (diameters) of the circles that may obey, with the help of a gnomonic approach (the "gnomonic factor"),
26 an algorithm involving the number of the circle and the height of the gnomon. The first assumption is that the
27 smallest radius would be obtained from the shadow of a gnomon on winter solstice at noon; the others by the
28 addition to the first a multiple of 1.5 times the gnomon's height. The members of the Association "Vla?i?i" for
29 Archaeoastronomy and the Archaeoastronomical Society from Romania, made the trip to Vr?ac. The circular
30 pattern they wanted to visit are five circles which centre is located at 45° 12' 37" N latitude and 21° 17' 14"
31 E longitude. This unusual earthwork dated around five millennia ago can be seen in Figure 1. The aim of the
32 trip was to exchange experiences and to formulate a joint work between both academic groups there. They had
33 troubles to make the walk through the ancient Big Swamp of Vr?ac; after reaching the edge of the largest circle,
34 the soil was not flat any more, and they can identify the five concentric circles: the larger with about 150m in
35 diameter; the inner one just 50m or less. Several things were reported about this trip [1]. However, at the SEAC
36 2018 Conference that took place in Graz, Austria, part of the group met Raul, with whom they discussed some
37 features on the collaboration done. The discussion arrived to the gnomonic factor [2] from which him exposed
38 the importance to consider the shadows of a gnomon at solstices (winter and summer).

39 The present report covers one aspect of the collaboration that arise when exchanging data about Vr?ac Circles;
40 the studies on the application of gnomonic factors (fg and fgp) to this site and the findings on the dimensions of
41 the circles and their interpretation as a possible Calendrical Monument.

42 In the first part, a search for the significant values of the gnomonic factors is done: two main proposals come
43 about when analysing the data. Then a study of the sizes (diameters) of the circles is made, discovering a special
44 regularity. From this behaviour of radius and diameters, a possible gnomonic algorithm came apparent and its

45 principal characteristics are presented in that section. These allow us to make some considerations on the possible
46 origin and objectives of the circles, correlating them with a calendrical division of the year.

47 2 Possible Gnomonic Algorithm & Calendar from

48 From that meeting and the discussion that took place a possible collaboration become feasible. The fact of
49 Latitude and its value of around 45° in the region of Serbia and Romania came later; the sites presented at the
50 SEAC 2018 Conference by us would make relevant the issue because gnomons' shadow would be equal to its
51 height. One of us (Marc) presented considerations about Sun alignments in the Neolithic times [3].

52 3 II.

53 4 The Gnomonic Factor search

54 The first step towards the evaluation of the possible influence of the gnomonic factor, fg , when the circles were
55 designed, was to consider several suggested values for the Obliquity [1]. In Table 1, Gnomonic Factors at Vr?ac,
56 the value of the factor expressed in decimal and as a fraction, are presented. As it can be seen easily, to values
57 become of significant importance: $2 \frac{1}{4}$ and $2 \frac{2}{7}$.

58 In the last two rows of the table, the exact value of obliquity has been defined to obtain the exact decimal value
59 of the fraction. In the previous to last row the exact value of 2.2500 or $2 \frac{1}{4}$, for fg appears: The corresponding
60 value of 24.0260 for obliquity implies an epoch of 3056 BC (using the Laskar algorithm to estimate it [4]). Then,
61 our first proposal is that around the end of fourth millennia BC, the selection of the site to make the earthwork
62 was made; specifically, a date around 3056 BC.

63 Due to the importance of the shadows of solstices at consideration the design of the circles could have took the
64 observations of shadows. Here, a second consideration can be made. The definition of the radius of the smallest
65 circle, the one which must have been defined in the first place, could arise from the measurement of the shadow
66 of a gnomon at winter solstice (WSs). A measurement using Google Earth ??5] gave us a diameter of 45.08m;
67 thus a radius of 22.5405m could be assumed. In Table 1, a gnomon of 8.546m is suggested in order to get a WSs
68 of that long. Then, our second proposal is to consider a possible gnomon of 8.546m height located at the centre
69 of the circles.

70 5 a) The Circles of Vr?ac

71 In a similar way, we have measured the diameter of the other four circles. The results of this operation are
72 presented in Table 2, Diameters of Vr?ac's Circles, and the ratios between them and the smallest are reported
73 also. One significant fact that must be enhanced here is the very similar values of the differences between
74 successive diameters. The next step in our analysis was to consider the estimation of the ideal radius for each
75 circle while considering the fractions reported in the last table, Table 2. In the following table, Table 3 Ideal
76 Radius and Proportions for Circles, we recover the results for the analysis that can be summarized as follows: a)
77 Five circles considered. We define the radius of the first circle as basis and numbered all of them from 1 to 5;
78 b) Ideal radius. Using the fractions reported in Table 2, we calculate the ideal values of the circles going from
79 22.5405 to 72.1280; c) Differences. We calculate the differences between the radius and the corresponding one of
80 first circle; and, d) Gnomon's fraction. We calculate how many times the gnomon height is in those differences.

81 Volume XVIII Issue II Version I The results appear very suggestive. The gnomon's fractions seem to be very
82 regular: 1.5, 3.0, 4.5 and 6 times the gnomon, approximately. They could reflect a specific algorithm defined by
83 the builders of the monument. Let's look forward to find a kind of procedure, possible, used around five millennia
84 ago.

85 6 b) Gnomonic Algorithm

86 An analysis of Table 3, allows us to define the main characteristics of the algorithm that possibly was used to
87 the design of the circles of Vr?ac. The radius of the circles seem to be equally separated and they appear to have
88 a relation with the length of the gnomon.

89 7 c) Definition 1.Vr?ac's Algorithm

90 Using a gnomon, g , at Vr?ac, the radius, r_n are the radius of the circles numbered with $n=1, 2, 3, 4$ and 5. The
91 radius are calculated with the following expression: $r_n = r_1 + (n-1)*b*g$ (1)

92 Where r_1 is the WSs = 22.5405m and b is a constant equal to 1.5 or $1 \frac{1}{2}$, $g = 8.546m$.

93 With this definition we have drawn the circles using GeoGebra [6] and are shown in Figure 2. In it, a colour
94 has been used in order to distinguish between them and for the reasons each of them will be apparent in the
95 following section. In Figure 3, the five circles are shown overlapping the Google Earth image of the monument
96 under analysis. It could be confirmed this way that the correspondence between the image and the drawing is
97 almost in perfect harmony to the north-eastern quadrant of the figure.

98 Then, a new proposal could be given: it is possible that the design of the Vr?ac's Circles has been based on a
99 simple algorithm including the use of a gnomon and the gnomonic factor.

100 8 On the image of Vrsac' Circles, Serbia

101 As it can be seen in the figure, there are marked in the south-north axis of the circles small points on the same
102 colour as the circles. The Blue one corresponds to r 1 ; the light blue circle to the l-blue point, etc. This approach
103 has to do with the other consideration appearing in Table 1: the size of the summer solstice shadow (SSs) that
104 reaches the length of 3.3120m (red point). In next section, we suggest a further interpretation of the already
105 presented results, in order to assimilate them in a calendrical context.

106 9 e) A Calendrical Design?

107 The Vr?ac's Circles is a well preserved monument. The locality has not been archaeologically explored. Only a
108 shallow excavation was done and materials found suggested that the site was occupied at Neolithic times [1], our
109 data goes in that direction for the dating. However, the probe on the ground has not been even clogged, gagging
110 like a wound in the central circle.

111 Although not protected by anything, the formation of concentric circles is in a surprisingly good condition.

112 It seems to be protected by the remote location; the salty soil here is unsuitable for farming and the quick way
113 of life of today's people no longer let them have time to walk around. Until about a hundred years ago it was
114 protected by a swamp, now drained by relatively shallow canals. In spite of drainage, in several places it can be
115 seen that the level of groundwater is high. In addition to this, the size of formation speaks in favor of Neolithic
116 dating (similar Bronze Age circular formations are generally smaller).

117 Assuming a possible Neolithic design, we can ask about the reasons to build such a set of rings. Has it a
118 relation with the religious ideas of the people? Or, was it more related to more quotidian issues like farming or
119 chasing?

120 Our approach to the circles have found a relation between the winter solstice shadows of a possible great
121 gnomon at the center. Then one can suggest that a kind of relation must be present for the other extreme
122 shadow of the year: the summer solstice shadow.

123 10 f) Definition 2. Vr?ac's Shadows

124 The specified gnomon shadows (Gs) could be defined taking the differences in length of the solstices shadows
125 (WSs, SSs, respectively) divided by 6 (g') (in a similar way as the circles), and then multiply by the fraction
126 considered (b = 1 ½); i.e., s 1 = WSs = 22.5405m;

127 (

128 The results are shown in Table 4.

129 In the table we have calculated other values that will be discussed next. The fifth column of the Table 4,
130 Sun_h (Sun's altitude), was calculated assuming that a gnomon (g = 8.546m) has to make a shadow of the
131 corresponding length (Gs). These lengths are represented as coloured points in the already discussed Figure 2.
132 These shadows appear in Figure 3. We have arrived to a kind of division of the year using the height of the Sun
133 and the corresponding shadows of the gnomon. Using a sky simulator such as, for example Stellarium [7], one
134 can verify the dates when the corresponding altitudes of the Sun were observed in the past. After an estimation
135 of the #day and the lapses between each marked day, we can define a possible Vr?ac's Calendar with the dates
136 of the year defining the principal days of a calendrical analysis. A Climatological analysis coming back to three
137 millennia BC would help to feed with specific conditions the possible origin of this Vr?ac's Calendar and correlate
138 them with the activities made by the people that constructed it. The colour of our circles in the drawing become
139 meaningful: Blue for winter; light blue for cold/ice; Green for spring; orange for hot; and, Red for summer.

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141 12 III. Discussion and Conclusion

142 Our discussion on the definitions made around the design of the five circles found at Vr?ac's site give us enough
143 evidence to say that a kind of gnomonic design was involved there. The evidence pointing to the existence of an
144 algorithm give us a tremendous task to try to confirm or disconfirm it. The mathematical abstraction that such
145 a thinking implies could give support to other ideas about Neolithic world that have been suggested before [8].

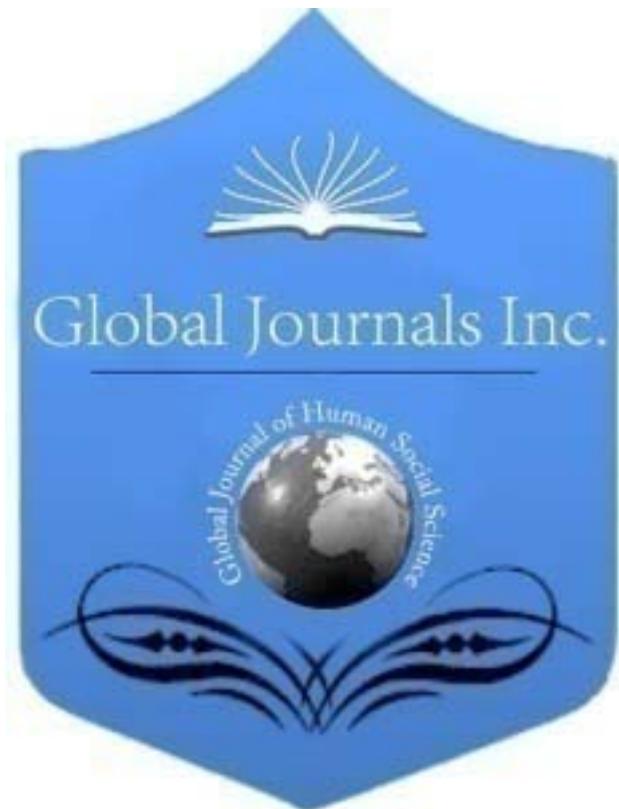
146 Maybe a gnomon of 8.546m would be too high to set in position in the first place. Today, we have not any
147 evidence of a work-earth at the site. Also, one must to assume that previous observations have to be made before
148 the structure was made. Table ??, Scaling the Gnomon, shows that if a 1.899m gnomon is set at the desired
149 point, the difference between solstices shadows is half the height of the final gnomon (obtained by us). A more
150 credible size for the gnomon that possibly the people of Vr?ac (Vinca culture [9]) used to select the site and
151 define the sizes of the circles.

152 13 Table 5: Scaling the Gnomon

153 This means that a gnomon of length 1.899m can explain the whole data of the site. Dividing this length by 6
154 one can expect to have a 0.3165m unit; maybe a unit called foot. The problem to this approach is that such a
155 unit and a gnomon would not left any evidence for the long period that have elapse since.

13 TABLE 5: SCALING THE GNOMON

156 Finely, we can suggest that the last table could confirms, precisely, that the site have been carefully selected and
157 that the design obeyed a kind of gnomonic criteria; i.e., there was while designing the Vr?ac's Circles, a gnomonic
158 algorithm that took into account the difference between solstices' shadows, more specifically, the possible use of
159 the gnomonic factor became one of its foundations. Such an idea can contribute to the development of more
detailed history of mathematics for that period. ¹



1

Figure 1: Figure 1 :

160



Figure 2:

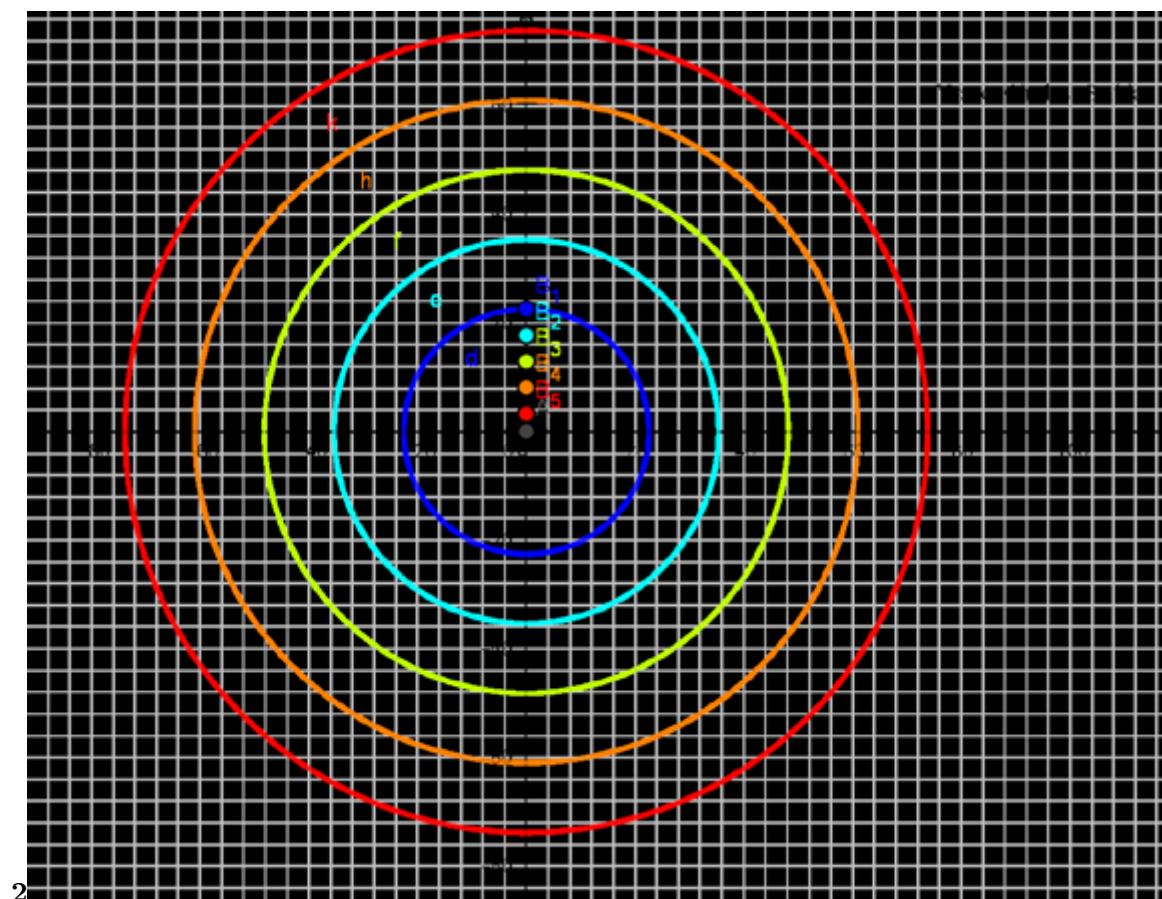


Figure 3: Figure 2 :



Figure 4: Figure 3 :

13 TABLE 5: SCALING THE GNOMON

1

Vr?as Serbia

| Lat | Lon | Epoch | Gnomon | 8.546 | Fg | fg' | fgp | WSc | SSc | |
|----------|----------|-----------|----------|----------|----------|--------|--------|---------|---------|--------|
| 45° | 12? | 21.287244 | 3056 | BC | | | | | | |
| 37? | o | | | | | | | | | |
| 45.21029 | 12.61758 | 37.0548 | | | | | | | | |
| 90 | | | | | | | | | | |
| Epo | Oblqty | WS | SS | WSs | SSs | Fg | fg' | fgp | WSc | SSc |
| 24.200 | 20.5897 | 68.9897 | 2.661909 | 0.384070 | 2.2778 | 2 2/7 | 1.6619 | 22.7487 | 3.2822 | |
| 24.150 | 20.6397 | 68.9397 | 2.654890 | 0.385072 | 2.2698 | 2 1/4 | 1.6549 | 22.6885 | 3.2908 | |
| 24.100 | 20.6897 | 68.8897 | 2.647861 | 0.38607 | 2.2618 | 2 1/4 | 1.6479 | 22.6286 | 3.2993 | |
| 24.050 | 20.7397 | 68.8397 | 2.640887 | 0.387077 | 2.2538 | 2 1/4 | 1.6409 | 22.5690 | 3.3079 | |
| 24.000 | 20.7897 | 68.7897 | 2.633944 | 0.388081 | 2.2459 | 2 1/4 | 1.6339 | 22.5097 | 3.3165 | |
| 23.950 | 20.8397 | 68.7397 | 2.627033 | 0.389086 | 2.2379 | 2 1/4 | 1.6270 | 22.4506 | 3.3251 | |
| 23.900 | 20.8897 | 68.6897 | 2.620153 | 0.390091 | 2.2301 | 2 2/9 | 1.6202 | 22.3918 | 3.3337 | |
| 23.850 | 20.9397 | 68.6397 | 2.613310 | 0.391100 | 2.2222 | 2 2/9 | 1.6133 | 22.3331 | 3.3421 | |
| 23.800 | 20.9897 | 68.5897 | 2.606489 | 0.392103 | 2.2144 | 2 2/9 | 1.6065 | 22.2752 | 3.3509 | |
| 23.750 | 21.0397 | 68.5397 | 2.599703 | 0.393110 | 2.2066 | 2 1/5 | 1.5997 | 22.2171 | 3.3595 | |
| 23.700 | 21.0897 | 68.4897 | 2.592947 | 0.394118 | 2.1988 | 2 1/5 | 1.5929 | 22.1593 | 3.3681 | |
| - | 24.0260 | 20.7637 | 68.816 | 2.637550 | 0.387559 | 2.2500 | 2 1/4 | 1.6376 | 22.5405 | 3.3120 |
| 3056 | | | | | | | | | | |
| - | 24.2330 | 20.5567 | 69.0227 | 2.666573 | 0.383410 | 2.2832 | 2 2/7 | 1.6666 | 22.7885 | 3.2766 |
| 7080 | | | | | | | | | | |

Figure 5: Table 1 :

2

| Diameter | Ratio | ratio_f | Diff |
|----------|------------|---------|-------|
| 45.08 | 1 | 1 0 | 27.37 |
| 72.45 | 1.60714286 | 1 3/5 | 25.05 |
| 97.50 | 2.16282165 | 2 1/6 | 23.08 |
| 120.58 | 2.67480035 | 2 2/3 | 23.24 |
| 143.82 | 3.19032831 | 3 1/5 | |

Figure 6: Table 2 :

3

| Circle Fraction | Ideal radius | Diff2 | Gnomon_f |
|-----------------|--------------|-------------------|----------|
| 1 | 22.540 | 0.000 | 0 |
| 2 | 36.064 | 13.524 1.58249473 | |
| 3 | 48.837 | 26.297 3.07707309 | |
| 4 | 60.107 | 37.567 4.39581871 | |
| 5 | 72.128 | 49.588 5.80248069 | |
| g | 8.546 | | |

Figure 7: Table 3 :

4

| Shad | Frac | Gs | Season | Sun_h | Day | #Day | Lapse | Calendar |
|------|------|---------|----------|--------|-------------|------|-------|----------|
| 1 | 1 0 | 22.5405 | Winter | 20.764 | 12/01/-3056 | 0 | | 21/dec |
| 2 | 1 | 17.2964 | cold/ice | 26.294 | 20/02/-3056 | 39 | 39 | 29/jan |
| | 1/2 | | | | | | | |
| 3 | 3 0 | 12.3436 | Spring | 34.696 | 18/03/-3056 | 65 | 26 | 24/feb |
| 4 | 4 | 7.9735 | | 46.985 | 20/04/-3056 | 98 | 33 | 29/mar |
| | 1/2 | | | | | | | |
| 5 | 6 0 | 3.3121 | Summer | 68.816 | 17/07/-3056 | 186 | 88 | 25/jun |
| g' | | 3.2366 | | 46.985 | 10/10/-3056 | 271 | 85 | 18/sep |
| | | | | 34.696 | 09/11/-3056 | 301 | 30 | 18/oct |
| | | | | 26.294 | 05/12/-3056 | 327 | 26 | 13/nov |
| | | | | 20.764 | 12/01/-3055 | 365 | 38 | 21/dec |

Figure 8: Table 4 :

161 .1 Acknowledgements

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