

# Theoretical Growth Model Space for Cities

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## Abstract

This work raises a theoretical model of spatial growth for the cities in which makes explicit that the growth of population and income growth determined the city's expansion in terms of area built for business and the area destined for housing. At the same time, shows that local public policy can slow down the growth of the population in the cities by avoiding that generate an unbalanced growth and thus increase the housing deficit.

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**Index terms**— City, dynamic urban, growth balanced space, population.

## 1 Introduction

The purpose of this paper is to propose a model of balanced spatial growth of cities. To this end, the paper is divided into five sections of which the first is this introduction. The second point is a brief presentation of the theoretical models of spatial growth while in the third and fourth presents the proposal of a balanced growth model together with its development and policy implications. In the fifth section draws the main conclusions.

## 2 II. Theoretical Proposals Growth Urban Space

The urban spatial growth can be understood as physical growth of the city and its people. Physical growth referred to the increase of housing, infrastructure and buildings for business within the urban area, while the population grows this accounted for as the rate at which expands people living within the city. Clearly this process can be understood as the dynamic behavior of the city. Understand what are the factors explaining this dynamic city, is what is known as a model of urban dynamics (MDU).

One of the first proposed to explain urban dynamics is based on the land transport systems. These systems are critical to the economy of a city and its planning because they can affect the location of families or households, the location of firms or companies, the paths of trade and because the levels of production and employment.

Micro theory is based on the individual consumer or consumers and was initially developed by Von Thunen. In this model we introduce transportation costs and their impact on the location and prices. After you were making changes to improve it to the point that their solution is to maximize the usefulness and benefits and of course in land rents.

Author : Ph.D. in Economics sciences of the National University of Colombia. Professor at the Santo Tomás and la Salle Universities. E-mail : josebernal@unisalle.edu.co; josereyes@usantotomas.edu.co Krugman (1997) (Krugman, 1999) has driven research in urban and regional context with the main analysis focuses on explaining the emergence, development and collapse of urban structures from the principle of self organization.

In the past years has produced a series of works related to the study of urban dynamics. Examples are the works cited by Ontiveros, Cirelli and Cavdeville. In this vein, the work done by Batty (1995), Batty (1995), Batty (2004), Batty (2005), White and Engelen (1993), Clarke, K., S. Hoppen and L. Gaydos (1997). These works rescue the implementation of models to explain urban dynamics based on cellular automata theory. These cellular automata classified urban space into cells which may contain specific areas of trade, industry or housing or because of specific social groups.

The models developed have allowed the explanation of the growth of cities and focused on the processes of structuring of urban space and the quest for development.

Likewise, other applications of these cellular automata models relate to the dynamics of the real estate market and urban ecology. Particularly Wegener and Spiekermann (2005) used cellular automata to model the diffusion process of urban pollutants. Additionally, the cellular automata theory has been applied to different urban transport systems as shown by Nagel and Schreckenberg (1992).

Other developments in urban dynamics models have been developed by Forrester (1969). This development tries to find the internal forces that control the balance between population, housing and business structures in urban areas.

The following is a specific growth model designed to Bogotá was originally based on urban dynamics models proposed by Forrester (1969) but including a lot of changes. The original idea of the model is to show the factors that control a hypothetical balance between population growth, housing and business. Therefore, this model will try to show the factors that affect the physical growth of the city along with population growth and the possible interactions that may occur to examine the balanced growth of the same.

### 3 a) Urbangrowth model

This model follows the logic of the city's growth is determined by population growth, the growth in physical units for housing and the growth of physical units designed for business. It starts with the following assumptions: a) It is assumed that there is a certain area of the city and part of that area can be designed to increase the supply of housing or facilities whether they be commercial or industrial. b) It is considered that the remainder of the total area for other uses but are essentially determined by the local government such as parks, gardens, roads, public space and infrastructure in general. Based on the above factors arise strict population growth, growth of housing and business growth.

### 4 Population growth

The population growth (Gn) of the city is directly related to the net growth rate of births (Gnac) with the growth rate of immigration (Ginm) and inversely with the growth rate of emigration (Gemi). In formal terms would have the following:  $Gn = f(Gnac, Ginm, Gemi)$  (1)

Also, the growth rate of immigration is directly determined by the employment rate of the city (Uu) for the wage gap between rural and urban (Ws) and other factors (Fs) in which are rural social conflicts, the demands of higher education and so on. These considerations have their theoretical support in the postulates of Todaro (1969), Lewis (1955) and Fay and Opal (2000). One of the initial relations that can be made between the process of economic growth and expansion of the city, created with the Lewis model of development which posed a structural change in production. He believed that in the rural labor was abundant with zero productivity which should migrate to the city where higher wages were paid and where they could train to participate in a mostly industrial area. The process should continue to generate a large industrialization to absorb the labor field, to the extent that such excess is exhausted and wages go up in the traditional sector. Todaro shown through a model of the impact of migration is generated in the labor market in the city. This whole process could involve an expansion or growth of the city both economically and in terms of space. As the region's population continues to increase, the pressures of urbanization increase and open spaces will be more appreciated and more expensive. Formally it would be:  $Ginm = f_1(Uu, Ws, Fs)$  (2)

Meanwhile, the net growth rate of births, which is the difference between birth rate and death rate depends on the rate of growth of income per capita (Gy) and the degree of education of women (Ged). It is possible that these two variables are correlated, but it is also true, on the theory of endogenous growth in fertility to the extent that women are brought up the possibility of having a large number of children is also reduced. It is also possible to find that with higher income levels is possible to increase the birth rate or otherwise access to higher education and vice versa. Formally have the following:  $Gnac = f_2(Gy, Ged)$  (3)

Under the reconsiderations it is then that the rate of urban population growth is an inverse function of the emigration rate and directly related to urban employment, wage differences, socio economic factors, political and cultural etc. of income per capita and level of education of women. Formally have the following:  $Gn = F(Gemi, Ws, Uu, Fs, Gy, Ged)$  (4) (-) (+) (+) (?) (?) (-)

Signs in parentheses indicate the relationship between each variable and the rate of population growth. It is noted that the impacts of other factors and the rate of growth of income per capita on the rate of population growth are not clear on the considerations above.

### 5 Physical growth in developable space

As noted above, the total urban area is the sum of space allocated for the expansion of the city in terms of housing construction in the construction of facilities for business, plus the area for the city's infrastructure and reserved area by the local government for different uses. Simplifying this division, the urban area is the sum of developable area and the residue for which local government provides. In formal terms can have the following:  $Au = Aurb + AUR$  (5)

From where  $Au$  is the total urban area,  $Aurb$  is the area  $Aurb$  and  $AUR$  is developable and the remaining area of the city designed specifically according to the guidelines and central government planning. If we divide equation 5 for  $Au$  must be developable are a involving more participation from the rest of the area of the city show 100%

of urban space. The task now is to show the factors that determine the physical expansion of developable space, expansion in the sense of the occupation of this space, both for use in housing construction and for construction of facilities for the industry or trade. Therefore arises as follows:  $A_{urb} = A_{bv} + A_{bn}$  (6)

Where  $A_{bv}$  developable area is intended for housing and  $A_{bn}$  is intended for the construction of commercial and industrial establishments.

## 6 Expansion of built-up area for housing

Within the area of developable space, the space for housing may be determined by several variables that affect the expansion and occupation of this space. The first factor affecting the rate of population growth than others is not strictly necessary to consider the close and direct relationship. It is well

## 7 Theoretical Growth Model Space for Cities

Global Journal of Human Social Science known that a process of urbanization is accompanied by increases in population and increases in housing construction. Normally, in most countries and especially in the underdeveloped population growth is faster than the growth of housing, thereby causing housing shortage for homes. In this regard, the population growth punctuates the expansion of the housing and if there is a shortage of the same, will tend to which the expansion takes place faster.

An important factor in the expansion of the housing is credit constraints. This variable can be and indeed it is, the variable after the population growth could impact positively on the expansion of housing, ie greater availability of credit to higher demand for homeownership and therefore increased the constructed area.

Decisions made by local government on a given area can influence the expansion of built-up area for housing and likewise to reduce it. Additionally, the local government can make investments in any specific area thereby encouraging housing construction.

These factors affect the expansion of housing built area horizontally, but you can think of a vertical growth of the constructed area would condition a bit in the horizontal expansion. This variable may be referred to as the efficiency of utilization of the soil, so that to the extent that this factor increases land use horizontally is reduced. In formal terms would have the following:  $A_{bv} = f_3(G_n, C_d, T_{dh}, D_p, T_{es}, P_s)$  (7) In this function  $A_{bv}$  is the share of housing intended for developable area within the total city area,  $C_d$  is the domestic credit,  $T_{dh}$  is the rate of housing deficits,  $D_p$  are the decisions of local government policy and  $T_{es}$  is the efficiency rate of soil showing the vertical growth of the city and  $P_s$  is the price of the area designated for housing.

## 8 Expansion of the built businesses

Within this context of urban dynamics may be given multiple inter relationships between the variables being studied, for example, one can argue that the very process of population growth leads to the generation of goods and services essential to the survival and welfare. Under these considerations, it is possible to propose a direct relationship between the expansion of built area for the operation of business goods and services between the rate of population growth. Jones (1998) argues that population growth brings increased productivity because there are more individuals who can generate innovations that lead to the creation of new economic sectors and therefore the creation of new businesses. Also, changing economic conditions may cause a transformation of the space used for housing in an area for business, in this sense, there may be a rate of demolition of houses to be replaced by commercial and industrial establishments. Of course, this process will take place in all areas of the city, but in areas where there are certain kinds of economic changes together with a large mass of the population but mostly because the displacement of the population of outlying areas is expensive. Formally have the following:  $A_{bn} = f_5(G_n, T_{dv}, G_y, O_f)$  (8) In the equation 8  $A_{bn}$  is the ratio of built area for business within the total developable area,  $G_n$  is the rate of population growth,  $T_{dv}$  is the rate of demolition of houses to be transformed into business,  $G_y$  is the growth rate income that positively impacts the construction of new space for business and "of" are other factors that may affect the growth of built area business such as land prices, policy decisions of local government, etc..

## 9 b) Dynamics of the model

The basic idea of the model is the fact that any factor that affects the population growth alter either developable space for housing construction or construction business. For example, if you increase the level of education of women, then this will lead to a decrease in the rate of population growth by the negative effect on the net birth rate and therefore will reduce the pressures on the use of developable space. Undoubtedly, that because of the lack of planning in the early rapid expansion of the city, provided the rate of population growth resulted in a greater housing shortage. The model could show a balance where possible balanced growth among the population, housing and business. This could be illustrated through a graph that relates the population growth along with the occupation of the developable area of the city. It is necessary to note that it is just an exercise to illustrate the possible balance through curves that reflect the growth of population and developable space. For this purpose the three equations that reflect urban growth, but can be reduced to two since the developable space is the sum of space used for more housing used for business. Formally have the following:  $G_n = F(G_{mig}, W_s, U_u, F_s, G_y, G_{ed})$   $A_{bv} = f_3(G_n, C_d, T_{dh}, D_p, T_{es}, P_s)$   $A_{bn} = f_5(G_n, T_{dv}, G_y, O_f)$  Putting together the last

two equations could be expressed by the following hypothetical equilibrium chart (No.1) that relates the spatial growth along with population growth: (D D D D) Gn Gráfico No. 1 a Gn 0 c Aurb 0 b Aurb

At point b of the graph relates the total developable area of the city but this can be increased by local government regulations or because they increase efficiency in land use or decreases in the rate of population growth.

It is clear that the model can be used at the city or disaggregation by localities or blocks or specific sectors of the city. If you are thinking of implementing this model to the block level to be submitted only drawback is the lack of information on population per acre because the rest of the information would be available.

Overall and in every model is expected to have high explanatory power, which is simple, applicable to the information available, etc.. Urban growth patterns as those based on cellular automata are very complicated but mostly costly in time for calibration and money for the software to be acquired for simulation and implementation; example is a model applied to the city of St. Francis whose cost amounted to two million dollars because its calibration takes a little over two years. Therefore, it would not be feasible for the time adapt a model for urban growth like this, but it would be appropriate to start thinking about implementing it to be important benchmarks for policy decision-making planning even local government.

## 10 IV.

## 11 Determination of Model of Urban Growth

For calibration of the model of urban growth will take first the equations that make up the model and the analytical solution of the same. The equations comprising the model are as follows:  $Gn = f(Gnac, Gnm, Gemig)(1)$

$$Gnm = f_1(Uu, Ws, Fs) \quad Gnac = f_2(Gy, Ged)(2)$$

The solution of the first block of equations leads to the equation of population growth in terms of exogenous variables in parentheses and the signs reflect the expected relationship between the behavior of the variation in population growth and behavior of each of variables.  $Gn = F(Gemig, Ws, Uu, Fs, Gy, Ged)$

$$Abn = f_5(Gn, Tdv, Gy, Of)$$

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The reduced form of this model of urban growth in terms of population growth and physical expansion of the city may be reflected in the following equations:  $Grcd F G F Gemig Grcd F G F F F W F U F dG Grcd Y GY Grcd Y GY S FS S WS V V N ? + ? + ? + ? + ? + ? + ? + ? = 2 2 2 2 1 1 0 1$

Where the sign of the coefficients are as follows: Where the sign of the coefficients are as follows:  $GN F 3 > 0$ ;  $cd F 3 > 0$ ;  $tdh F 3 > 0$ ;  $Dp F 3 > / < 0$ ;  $TES F 3 < 0$ ;  $Gy F 5 > 0$ ;  $of F 5 < / > 0$ .

The last two coefficients are also positive because higher GDP growth of a city the greater the area designated for business.

The intention now is to try to find the coefficients more robust for making some kind of simulation in some cities that lead to consistent results in terms of population growth and growth in the use of the urban area whether for business or housing.

V.

## 12 Conclusions

We have proposed a theoretical model of spatial growth for cities in which it is made explicit that the growth of population and income growth determine the expansion of the city in terms of built area for business and the area allocated for housing. It also plays a key role local public policy to curb population growth in cities leads to unbalanced growth generating increases in residential housing shortage.



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Figure 1:  $AuAbv = f_3$  (



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215 [Theoretical Growth Model Space for Cities] , *Theoretical Growth Model Space for Cities*  
216 [Von Thunen ()] , J Von Thunen . *El Estado Aislado* 1820.  
217 [Krugman (ed.) ()] , Paul Krugman . Geografía y comercio, Antoni Bosch editor (ed.) 1992. p. 152.  
218 [Nagel and Schreckenberg ()] *A Cellular Automaton Model for Freeway traffic*, K Nagel , M Schreckenberg . 1992.  
219 (in Journal physique I)  
220 [Batty ()] 'A chronicle of scientific planning -The Anglo-American modelling experience'. M Batty . *Journal of*  
221 *the American Planning Association* 1994. 60 (1) p. .  
222 [Todaro ()] 'A Model of Labor Migration and Urban Unemployment in Less Developed Countries'. Michael P  
223 Todaro . *American Economic Review* 1969. 59 p. .  
224 [Clarke et al. ()] *A selfmodifying cellular automaton model of historical urbanization in the San Francisco Bay*  
225 *area, Environment and Planning B: Planning and Design*, K Clarke , S Hoppen , L Gaydos . 1997. 24 p. .  
226 [Batty ()] 'Agents, cells and cities: new representational models for simulating multiscale urban dynamics'. M  
227 Batty . *Environment and Planning A* 2005. 37 p. .  
228 [Alberti and Waddell ()] 'An Integrated Urban Development and Ecological Simulation Model'. M Alberti , P  
229 Waddell . *Integrated Assessment* 2000. 1 (3) p. .  
230 [White and Engelen ()] 'Cellular Automata and fractal urban form: a cellular modeling approach to the evolution  
231 of urban land-use patterns'. R White , G Engelen . *Environment and Planning A* 1993. 25 p. .  
232 [Couclelis ()] 'Cellular worlds: a framework for modelling micro-macro dynamics'. H Couclelis . *Environment and*  
233 *Planning A* 1985. 17 p. .  
234 [Batty ()] *Dissecting the streams of planning history: technology versus policy through models, Environment and*  
235 *Planning B: Planning and Design*, M Batty . 2004. p. .  
236 [Krugman (ed.) ()] *La organización espontánea de la economía*, Paul Krugman . *The selforganizing economy*,  
237 Antoni Bosch editor (ed.) 1997. p. 167.  
238 [Spiekermann and Wegener ()] 'New timespace maps'. K Spiekermann , M Wegener . *Proceedings of the 4th*  
239 *International Conference on Computers in Urban Planning and Urban Management*, R Wyatt, H Hossain  
240 (ed.) (the 4th International Conference on Computers in Urban Planning and Urban Management Melbourne.  
241 Melbourne) 1995. 2 p. . Department of Geography and Environmental Studies, University of Melbourne  
242 [Krugman et al. ()] 'On the evolution of hierarchical urban systems'. Masajisa Krugman , Fujita , Mori . *European*  
243 *Economic Review* 1999. 43 p. .  
244 [Jones ()] 'Population and Ideas: A Theory of Endogenous Growth'. I C Jones . Inc. 46. *Departments of*  
245 *Economics. NBER Working Papers* 1998. 6285. (National Bureau of Economic Research)  
246 [Wegener et al. ()] 'Sustainable Lifestyles? Microsimulation of Household Formation, Housing Choice and Travel  
247 Behaviour'. M Wegener , P Waddell , I Salomon . *Social Change and Sustainable Transport*, W Black, P  
248 Nijkamp (ed.) (Bloomington, IN) 2002. Indiana University Press.  
249 [Berry and Garrison ()] 'The functional Bases of the Central Place Hierarchy'. Brian Berry , Willian Garrison .  
250 *Readigs in Urban Geography, Mayer and Kohn editores*, 1958. The University of Chicago Press. p. .  
251 [Von Newmman ()] *Theory of selfreproduction automata*, J Von Newmman . 1966. University of Illinois Press.  
252 [Forrester ()] *Urban Dynamics*, J Forrester . 1969. Cambridge, MA: MIT Press.  
253 [Fay ()] *Urbanization without Growth: A Not-so-Uncommon Phenomenon*, M Fay , C . 2000. Washington DC,  
254 World Bank Policy Research WP 2412.  
255 [Waddell ()] 'UrbanSim: Modeling Urban Development for Land Use Transportation and Environmental  
256 Planning'. P Waddell . *Journal of the American Planning Association* 2002. 68 (3) p. .